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The Law of Scale Independence

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ABSTRACT

Geography and geosciences deal with phenomena that span spatial scales from the molecular to the planetary, and temporal scales from instantaneous to billions of years. A strong reductionist tradition in geosciences and spatial sciences tempts us to seek to apply similar representations and process-based explanations across these vast-scale ranges, usually from a bottom-up perspective. However, the law of scale independence (LSI) states that for any phenomenon that exists across a sufficiently large range of scales, there exists a scale separation distance at which the scales are independent with respect to system dynamics and explanation. The LSI is evaluated here from five independent perspectives: geographic intuition, dynamical systems theory, Kolmogorov entropy, hierarchy theory, and algebraic graph theory. All of these support the LSI. Results indicate that rather than attempting to identify the largest or smallest relevant scales and work down or up from there, the LSI dictates a strategy of focusing directly on the most important or interesting scales. An example is given from a hierarchical state factor model of ecosystem responses to climate change.

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1. Introduction

Explanation in geography and geosciences must confront features and processes ranging from molecular to planetary, and from instantaneous to billions of years (Figure 1). The strong reductionist tradition in science seeks to apply similar representations and process-based explanations across these vastscale ranges, usually from a bottom-up perspective. This is despite the fact that abundant empirical evidence, supported by multiple theoretical perspectives, indicates that process-response relationships, and appropriate representations of them, vary with scale. In this paper several of the latter are reviewed and synthesized to propose and test the *law of scale independence* (LSI). The LSI asserts that in geographical systems processes and controls that operate at sufficiently different spatial and/or temporal scales are independent of each other with respect to their effects on system function and evolution.

The aphorism *everything is related to everything else, but near things are more related than distant things* is often posed as the First Law of Geography. The second part of the law relates to spatial autocorrelation and distance decay. The LSI is analogous in that it relates to what might be called scale autocorrelation (relatedness across scales rather than distance), and distance decay, with distance conceptualized as the difference between scales. LSI can also be perceived as a scale-domain version of a putative principle of geographic similarity, whereby similarity of spatial configurations is proportional to the similarity of underlying processes.

The purpose of this paper is to examine the LSI from multiple perspectives. Before launching into those arguments and illustrations I will provide some background on the relevance of the topic, and on previous efforts to address scale independence.

1.1. *Scale linkage and scale contingency*

In the geosciences and geographical information science there are three broad, overlapping categories of scale problems. The first, *resolution*, concerns the level of detail necessary to properly observe and represent a given phenomenon, and tradeoffs among sampling effort, sizes of data sets, and analysis and computational times.

Scale linkage is the second major issue, referring to the problems of linking representations along the often-vast range of applicable scales. In ecohydrology, for instance, how do we link processes operating in individual leaf stomata to biome-scale patterns of evapotranspiration, both of which we know are important? How do we link the mechanics of flow acting on a sand grain in a streambed to evolution of fluvial landscapes? Scale linkage has operational and theoretical aspects.

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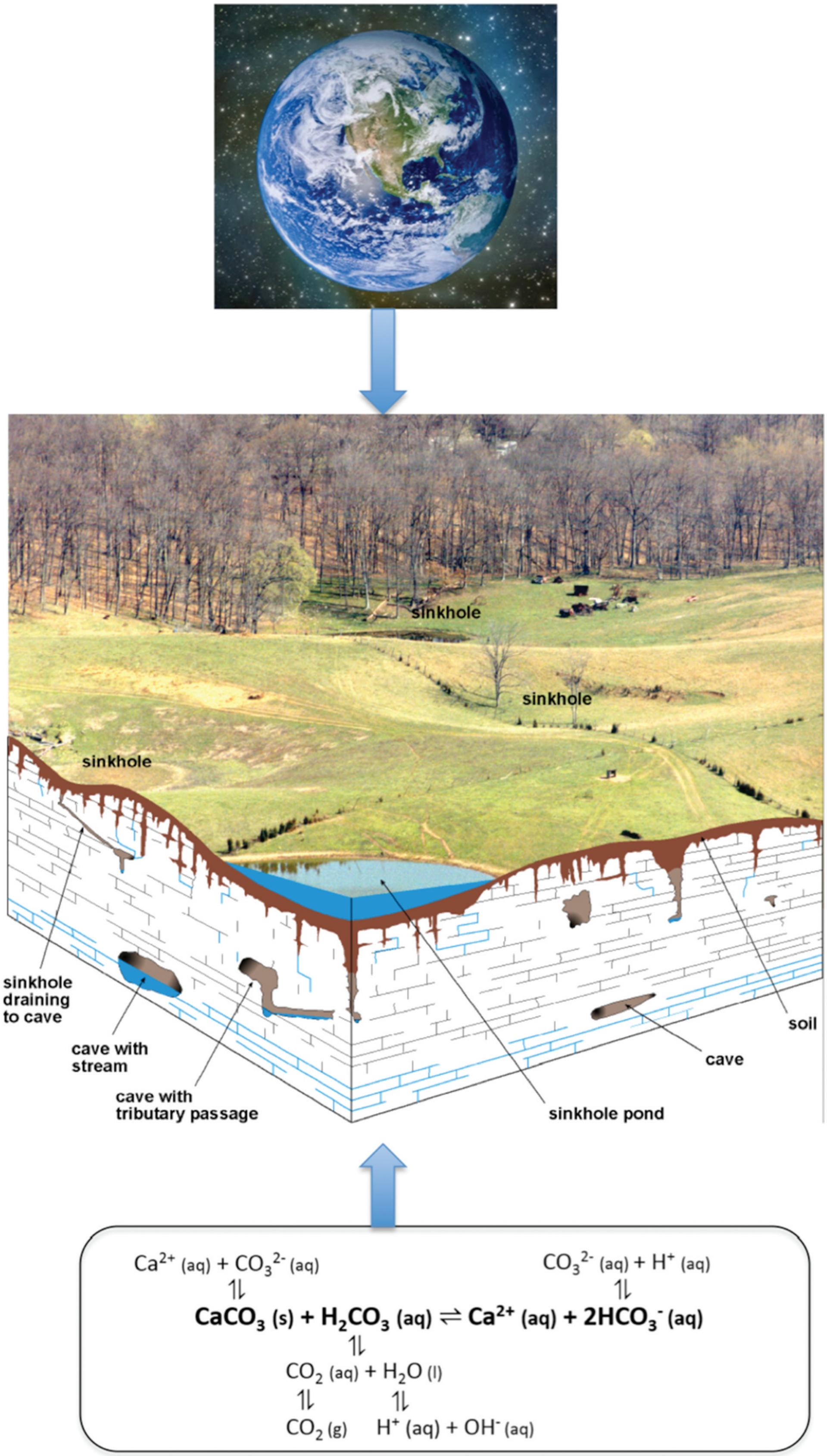

Figure 1. The karst landscape in Estill County, Kentucky (as depicted by the Kentucky Geological Survey), like most geographical features, is affected by multiple scale causality. From the bottom, spatial scales as small as the molecular and as rapid as rates of chemical reactions and water flow are important. From the top, scales as large as global climate and as slow as geological evolution are important.

The former involve identifying characteristic scales, determining conditions of scale independence, decisions as to what phenomena to represent at a given scale, and tools for transferring information and representations between scales. Theoretical scale linkage issues involve fundamental questions of whether any single rules, relationships or representations are even potentially valid across all relevant-scale ranges; bottom-up vs. top-down influences, and multiple-scale causality. These are the main topics addressed in this paper.

Closely related is the third basic issue, *scale contingency*. If the principles governing process-response and spatial relationships were constant across scales, then scale linkage would chiefly be a technical issue. These problems, though quite challenging, are well known in the context of problems, such as multiple-resolution models, upscaling, and downscaling. In physical geography and geosciences, however, the rules are typically not constant across scales. Problems of interpreting Earth surface systems associated with scale contingency of the critical controls have been discussed for example, for coastal dunes (Sherman 1995; Kim and Zheng 2011), salt marshes (Kim 2018), coastal landscapes (Walker et al. 2017), prairie ecosystems (Zirbel et al. 2019), soil and hillslope hydrology (Ma et al. 2017; Glaser et al. 2019), soil-plant-water interactions (Manzoni et al. 2013), rock weathering (Viles 2001; Inkpen 2011), disturbance of soil landscapes (Sauchyn 2001), river bank erosion (Couper 2004), and drainage basin sediment budgets (Slaymaker 2006).

Problems of scale linkage have probably always been recognized, at least implicitly, in geography, geosciences and ecology. Broad, explicit consideration can be dated to 1965, when Haggett (1965) articulated the problem in geography, and Schumm and Lichty (1965) published their famous paper on the relationship between temporal scale and independence of variables in geomorphology. The significance of scale linkage is such that it has been described as geomorphology's 'holy grail' (Rhoads and Thorn 1996, 145; Couper 2004, 392).

In geosciences, the term 'scaling' has become strongly associated with power-law scaling and related phenomena. Scaling is in some cases linked to scale linkage, but it is difficult to generalize because power-law scaling is an example of equifinality – that is, it can be produced by several different processes or histories. Jiang (2017) reduced the implications of power-law scaling to its essence, terming it the scaling law: there are far more small things than large ones in geographic space, and asserted the scaling law as the dominant design principle for cartography.

In short, the scale independence treated here concerns the question of whether, with respect to a given phenomenon, processes at different scales are independent. In some studies of scaling, particularly those using fractal analyses, scale independence refers to self-similar statistical and topological properties across (a range of) scales. This form of scale independence is quite different from scale independence as defined here. Essentially, self-similarity implies that scale (resolution) has no effect on patterns – that is, their properties are similar (they 'look' the same) at any resolution at which self-similarity applies. The notion of scale (in)dependence examined here, by contrast, relates to processes, controls, and functional linkages rather than geometrical or topological patterns or statistical properties of spatial patterns or distributions. The latter may or may not be clearly linked to processes and controls, but in many studies of fractals and self-similarity the connection is ignored or assumed. For instance, numerous studies of stream networks using fractal analyses and other morphometric properties have been carried out. However, in most cases, the self-similarity and other properties identified are associated with any hierarchically ordered branching network and not necessarily related to any hydrological or geomorphological factors (e.g. Richards 1982, 35; Abrahams 1984; Kirchner 1993; Peckham and Gupta 1999; Zanardo, Zaliapin, and Foufoula-Georgiou 2013; Kovchegov and Zaliapin 2016). While fractal properties and other morphometric measurements can be and have been linked to environmental controls and hydrological and geomorphological processes, the morphometric indices by themselves (including self-similarity) are not physically meaningful.

1.2. Scale (in)dependence

If processes and controls operating at different scales are not independent, this suggests that a seamless linkage across those scales is at least possible. If the scales are independent, this can complicate scale linkage due to the need for application of multiple rules and representations at different scales. Or, scale independence could simplify the problem by justifying the focus on some and exclusion of other scales.

Few would argue that in a practical, pragmatic sense it is useful and valid to emphasize some and ignore other scales. Palaeoclimatologists, for instance, might in many cases ignore solving the atmospheric equations of motion, while dynamic climatologists would often have no need to consult paleoclimate evidence. But is scale independence a general principle with the force of a geographic or scientific law? Or perhaps just a convenient simplifying assumption?

Forriez, Martin, and Nottale (2020) argued against scale independence from a reductionist, geometrical perspective grounded in fractal geometry. They promote a continuum view of geographical scale (as opposed to a hierarchical concept), and argue for scale relativity, whereby every phenomenon is defined relative to a reference system that is itself relative. Their programme is to discover geographic laws that are applicable regardless of the reference system, and therefore the scale.

Geographical systems are networked, and arguing from actor-network theory, Allen (2011) questioned the whole concept of scale. If everything is networked, he claims, then scale as applied to any one landscape becomes irrelevant. Inkpen (2011) also takes a relational view of scale, rather than scale as part of a fixed, absolute reference point. However, his framework is geared towards identifying critical scales for analysis rather than making scale irrelevant.

In the context of Earth surface modelling using a combination of remote sensing and ground data, Yue et al. (2016) implicitly rejected scale independence, though they did not explicitly address scale linkage or contingency. Their fundamental theorem of Earth surface modelling is formulated on the basis of applying high accuracy surface modelling (HASM) to simulating surfaces. The theorem is that an Earth's surface system or a component of Earth's surface environment can be simulated with HASM when its spatial resolution is fine enough, which is uniquely defined by both extrinsic and intrinsic invariants of the surface. They derived seven corollaries, all based on upscaling, downscaling, and data fusion and assimilation. Thus, the implicit assumption is that the laws or principles operating at a certain (fine) scale are sufficient for all modelling purposes.

By contrast, the studies cited earlier all acknowledge or support a degree of scale independence (Sherman 1995; Viles 2001; Couper 2004; Slaymaker 2006; Inkpen 2011; Kim and Zheng 2011; Manzoni et al. 2013; Ma et al. 2017; Walker et al. 2017; Kim 2018; Glaser et al. 2019; Zirbel et al. 2019). To be sure, not all directly address scale independence as a law or general principle, or explicitly reject or argue against the possibility of seamless linkages across a large range of scales. But all, at least within the domains of study addressed, confirm that different rules or representations are sometimes necessary at different scales.

The analyses of scale independence that follow are organized by several different perspectives, arguments, and analytical approaches: intuition, dynamical systems theory, Kolmogorov entropy, hierarchy theory, and algebraic graph theory. They are, to varying degrees, based on my own efforts to address scale independence over the years, to see if the arguments converge towards a general principle of scale independence.

2. Evaluating scale independence

2.1. Intuition

Intuition is defined as the ability to understand something without conscious reasoning, or a thing that one knows or considers likely from instinctive feeling. It is intuitively evident, for instance, that while plate tectonics are relevant to landforms, tectonophysics is not useful to explain process mechanics at the scale of a hillslope or stream channel. Similarly, while we know that the physics of wind drag on a grain of sand is germane to dune dynamics, it is not helpful in studying Quaternary evolution of dune fields. Many other examples exist – ecosystem functions cannot be explained by cellular microbiology (or cell biology by ecosystem science), for instance.

While intuition is not highly valued as a type of formal testing, it is in some cases based on extensive experience and insight. Inkpen (2011, 10), for example, stated that Schumm and Lichty's (1965) argument with respect to temporal scales and geomorphology, that factors change from dependent to independent as scale changes 'is more a statement of conviction than a theory but one that has served geomorphology well'. A formal mathematical basis for Schumm and Lichty's arguments and an extension to spatial as well as temporal scale were provided later (Phillips 1986, 1988), but this had little or no role in their widespread acceptance by geomorphologists. The popularity and success of Schumm and Lichty's (1965) position is due to its consistency with common sense and geoscientific intuition, and also to a general respect for the value of the intuition of the authors.

Intuition also does not necessarily imply the absence of any formal logic or rigorous analysis in support of intuitive insights. At least at the far ends of the scale ranges that geography and geosciences deal with, intuition strongly supports scale independence. However, in many cases independence is far from intuitively obvious. For instance, in a modelling environment, consider an effort to model two phenomena that occur at different time scales (subscripts *s* and *f* for slower and faster). To ensure that changes are not propagated through space more rapidly than they actually happen, these conditions should be met:

$$\Delta tf \leq \Delta Sf/C \quad (1)$$

$$\Delta ts \leq \Delta Ss/m \tag{2}$$

Δt, ΔS represent the time steps and spatial resolution (e.g. grid spacing), *C* is the rate of the most rapid fast processes, and *m* of the least rapid of the slower processes. In the specific case of numerical modelling of partial differential equation systems, these conditions correspond to the Courant-Friedrichs-Lewy (CFL) condition, frequently used in climate models. Martin (1993) reviews CFL applications in the context of modelling vegetation/climate interactions, for example.

We can deduce from this that

$$\Delta Ss/\Delta Sf \leq (m/C)(\Delta ts/\Delta tf) \tag{3}$$

$$\Delta ts/\Delta tf \geq (C/m)(\Delta Ss/\Delta Sf) \tag{4}$$

For instance, if due to the use of a fixed spatial grid scale *ΔSs* = *ΔSf*, the time step for the slower processes should be longer than that of the faster processes by *C/m*. If *Δts* = *Δtf* due to use of a common time step, the grid size for the faster processes should be smaller than for the slower ones by *m/C*. If these criteria imply unrealistically large, small, or different grid sizes or time steps, spatial independence is indicated.

2.2. Dynamical systems

Geographical systems are often characterized as a set of interconnected, networked components and the interactions between them. The components may be processes, mass/energy storage components, or environmental factors or controls. Dynamical systems approaches are concerned with studying the system-level behaviour of these interconnected systems.

In the study of ecological systems (and other geographical systems) one is often obliged to abstract selected factors or variables from the overall, broader system. Schaffer (1981) was concerned with such ecological abstraction. He developed the product theorem for abstracted eigenvalues, in the context of a dynamical system characterized by two different groups of components, one of which operates at faster rates, shorter time periods, or more localized scales, and the other at slower rates, longer time periods, or broader spatial scales. The Jacobian matrix of the system can then be partitioned into four submatrices: the mutual influences of the fast/local components on each other; the mutual influences among the slow/broad scale components; effects of the slow on the fast components; and effects of the fast on the slow. The eigenvalues reflect the dynamical behaviour of the system. Eigenvalues of a subgraph of the abstracted components of a system are related to those of the larger, parent system by

$$\prod_{m=1}^{n(q)} \lambda_{m(a)} = \frac{\left[\prod_{j=1}^{N} \lambda_j\right]}{\left[\prod_{k=1}^{N-n(q)} \lambda_{k(d)}\right]} \tag{5}$$

$\boldsymbol{\lambda}_{m(a)}$ represents the eigenvalues of the abstracted system, $\boldsymbol{\lambda}_{\mathrm{j}}$ those of the parent system, and $\boldsymbol{\lambda}_{\mathrm{k(d)}}$ those of a subgraph consisting of the remaining components (not abstracted).

When subsystems *a, d* consist of components that operate at distinctly different scales ($a \ll d$ or $a \gg d$), Schaffer (1981) proved, as eq. (5) suggests intuitively, that components that operate at distinctly different scales are independent of one another in regard to their effect on system dynamics. The principle can be extended to multiple scale levels or levels of abstraction.

Thus, at least for the case of the dynamics of geographical systems that can be represented as nonlinear dynamical systems, the LSI is supported: if the scales are sufficiently different, they are independent. The abstracted eigenvalues theorem was used by Phillips (1986, 1988) to provide a formal basis for Schumm and Lichty's (1965) principles and to demonstrate independence of human agency and Holocene sea-level rise variables in assessing aggradation of lower coastal plain streams (Phillips 1997). The approach can, of course, also be used to show that phenomena at different scales are not sufficiently different to be independent, as Phillips (1995) did for the case of relationships between floodplain ecological dynamics and alluvial sedimentation. Other applications of the abstracted eigenvalues theorem are mainly in ecology, and include Kerfoot and DeAngelis's (1989) analysis of scale-dependent dynamics of ecological food webs, Auger and Benoit's (1993) study of predator-prey dynamics, and O'Neill, Kahn, and Russell (1998) on conservation ecology.

2.3. Kolmogorov entropy

While numerous subcategories exist, three broad entropy concepts are common in Earth sciences, geography, and ecology, deriving from thermodynamics, information theory, and nonlinear dynamical systems (NDS) theory. The NDS-based entropy concept, called Kolomogorov (K-) entropy, measures the divergence or convergence in state space of a nonlinear dynamical system. If a spatial pattern or temporal sequence is the outcome or a manifestation of a nonlinear dynamical system, then the K-entropy is the same as change of the statistical or informational entropy (Oono 1978; Culling 1988; Zdenkovic and Scheidegger 1989; Fiorentino and Claps 1992). For example, if the Shannon (information) entropy of a spatial pattern increases as the system evolves, K-entropy is positive, and vice-versa. Positive K-entropy is associated with dynamical instability and deterministic chaos (increasing variability over time), and negative K-entropy with dynamical stability. K-entropy is therefore a fundamental indicator of (spatial) system dynamics. If entropy switches between negative and positive at different scales, this is a clear indication of scale independence.

The arguments that follow are taken from Phillips (2005), who took a starting point from physics and generalized to the more complex systems of geography.

Envision a simple situation where a fundamental minimal-scale $\Delta_o > 0$ applies. A process operating at Δ_o becomes observable at a broader scale Δ, where $\Delta = N\,\Delta_o$. For example, in sediment transport studies the physical forces acting on a single grain are often considered the fundamental minimum scale. The processes at this scale become observable at the scales of, e.g. sediment transport and deposition, bedforms, and changes in channel or surface morphology. Entropy of the observed pattern $H(\Delta)$ is directly related to the entropy of the fundamental process (Boyarsky and Gora 2001):

$$H(\Delta) = N\ H(\Delta_o) \tag{6}$$

Equation (6), while appropriate for simplified physical systems, is inadequate for most spatial systems. Weather systems, landforms, soils, ecosystems, hydrological systems, and other Earth surface systems are generally influenced by multiple processes and controls that operate at various spatial scales. Generalizing from equation (6) to cases where multiple processes produce the observed pattern (initially assuming for simplicity that the observational scale is the broadest relevant scale), then at the observational scale the entire system entropy is additively composed of the entropies from the smaller-scale processes:

$$\underset{i=1}{\overset{m}{H}}(\Delta_{max}) \ = \ \Sigma[N_i H(\Delta_i)] \tag{7}$$

where there exist $i = 1, 2, \ldots m$ smaller-scale processes producing the system observed at Δ, and $N_i\ \Delta_i = \Delta$. N represents the scale ratio, $N_i = \Delta/\Delta_i$. These equations follow from the additive properties of entropy, and assume that the generating processes are independent. In geography independence is often not the case, so

$$\underset{i=1}{\overset{m}{H}}(\Delta_{max})\Sigma[N_i H(\Delta_i)] \tag{8}$$

Geographical systems, unlike the physics laboratory, are typically influenced by broader as well as smaller scale phenomena. Now assuming that the system is observed at the most detailed relevant scale (again, for simplicity), observed entropy is composed of scaled portions of the entropies of the broader scale properties (following from eq. 6 and the additive properties of entropy). As the $j = 1, 2, \ldots q$ broader scale controls may well not be independent,

$$H(\Delta_{min}) \sum_{j=1}^{q} [N_j H(\Delta_j)] \tag{9}$$

While $N_i > 1$, $N_j < 1$.

Combining equations (8) and (9):

$$H(\Delta) = \sum_{i=1}^{m} [N_i H(D_i)] + \sum_{j=1}^{q} [N_j H(\Delta_j)] \tag{10}$$

This assumes that each of the $m + q$ processes operates at a characteristic scale associated with N_i, N_j. Processes operating at >1 scale could be accommodated by treating them as separate processes at each operative scale.

All relevant processes and controls can often not even be identified, much less measured or estimated. For convenience, all broader-scale phenomena relative to the observational scale are denoted with subscript g for global, and finer-scale ones with l for local.

$$H_l < \sum_{i=1}^{m} \ [N_i H(D_i)] \tag{11}$$

$$H_g < \sum_{j=1}^{q} [N_j H(D_j)] \tag{12}$$

While assessing each process or control at all possible scale is often impossible or unfeasible, it is often possible to examine patterns at multiple scales and thereby estimate H_l, H_g.

From this point we replace $H(\Delta)$ with H_r to emphasize that the scale of observation may be only one of several possible, with H_r representing the entropy of a spatial system reflecting aggregate influences of local and global (smaller and broader relative to r) controls, with associated entropies H_l, H_g. Analogous to eq. (10), but with aggregate values for N, H:

$$H_r \leq \ N_l H_l + \ N_g H_g \tag{13}$$

For example, entropy of a soil map at the scale of interest could be H_r, and H_l, H_g the entropies of the soils mapped at more detailed and at broader scales (or in a GIS environment, aggregated at different resolutions) This assumes that the scales associated with l, g capture the relevant smaller and larger sources of variability. For the case of soils, H_g could be associated with regional variations and gradients in geology, climate, and topography, and Hl might reflect variations within fields or hillslopes, and gradients in drainage, vegetation cover, and microclimate. As an example, if the scale of the mapped pattern (r) is 1:24,000 and H_l is based on soils mapped at 1:600, then N_l = 40. If the scale of the 'global' map is 1:100,000, Ng = 0.24.

Based on the chain rule for entropy

$$H_r = N_l H_l + \left[\left(H_g N_g\right)|\left(H_l N_l\right)\right] = N_g H_g + \left[\left(H_l N_l\right)|\left(H_g N_g\right)\right] \quad (14)$$

where [(*HgNg*)|(*HlNl*)] and [(*HlNl*)|(*HgNg*)] indicate the global entropies as constrained by (or conditional on) the local entropies, and vice-versa. The mutual information (*MI*; reduction of uncertainty in either the local or global variables by knowing the other) is given by

$$MI = \left(N_l H_l + N_g H_g\right) - H_r \quad (15)$$

Phillips (2005) showed that this indicates an approach where entropy estimated or measured at r and either g or l allows estimation of the contribution from either smaller or larger scales by solving for the appropriate term in eq. (15), and applied it to a soil geography case study.

Notwithstanding the fact that not all m local or q global factors can necessarily be identified or separated, from eq. (10) we can write

$$H_r = \sum_{i=1}^{m}[N_l H_l] + \sum_{j=1}^{q}\left[N_g H_g\right] = \bar{N}_l \bar{H}_l\, m + \bar{N}_g \bar{H}_g q \quad (16)$$

Rearranging, we obtain

$$\bar{N}_l \bar{H}_l m^- > H_r - \sum_{j=1}^{q}\left[N_g H_g\right] \quad (17)$$

$$N_g H_g q^- > H_r - \sum_{i=1}^{m}[N_l H_l] \quad (18)$$

If H_r is fixed (no additional measurements or observations are made at that scale), we can examine the implications of identifying and including additional global or local scale controls (i.e. increasing m or q).

For the case of additional broader scale factors, $\Sigma [N_g H_g]$ can only increase, ultimately resulting in negative local entropy (a net source of information). Similar logic applies to additional local factors and $\Sigma[N_l H_l]$.

The sign of H is independent of the value of N, m or q, which must be finite positive. As long as the global (local) entropy contribution is positive, expanding (decreasing) the scale or the number of broader-scale controls decreases the local (global) entropy contribution, which, at some point, must become negative. Because observed entropy of a spatial pattern must be ≥ 0, negative entropy in this context indicates a source of information. This change in the direction of influence is termed a qualitative causal shift. An entropy change and causal shift of this type associated with scale or resolution is a clear indication that processes at the scales in question are independent.

This analysis shows that, as the range of scale is increased in either direction, either by broadening or narrowing resolutions or by incorporating more controls, the effect of larger/smaller scale influences not only changes, but may change qualitatively, for example, in terms of having positive (entropy-increasing) or negative (entropy-decreasing) effects. Expanding the scale not only modifies the relative importance of larger or smaller scale influences but may change whether they have the effect of producing order or complexity into the observed spatial pattern. That a causal factor or mechanism may have qualitatively different explanatory implications, depending on the range of scales considered, is consistent with scale independence.

The likelihood of qualitative causal shifts is reduced when there is a finite, and sometimes small, number of processes and controls that significantly affects a given observed spatial system. The range of scales that may influence the observation scale is also finite, and sometimes relatively limited. This is, in fact, the basis of hierarchy theory, discussed in the next section.

2.4. Hierarchy theory

Hierarchy theory (HT) is based on a nested structure of scales, and has no direct relationship with the notion of social, political, economic, or chain-of-command hierarchies. At level i in the hierarchy, patterns and dynamics are affected by factors and processes operating at that level, at one level above (coarser scale; $i + 1$), and at one level below (finer scale; $i - 1$). At two or more levels above or below i, factors operate either too rapidly or at too fine a resolution, or too slowly or at too coarse a scale, to be observed at i, or at least those effects are entirely mediated by intermediate hierarchical levels. HT is *not* a tool or framework potentially enabling seamless linkage across the entire range of relevant scales. Rather, HT implies that scale linkage must be stepwise; as one ascends or descends the 'scale ladder', new factors and processes become important and others cease to be relevant. Thus HT inherently and by definition indicates scale independence.

The hierarchical nature of scale in geography and geosciences is often implicit. Sometimes the hierarchies are unambiguous, as they are functional and spatially nested. This is the case, for instance, with the hierarchy of hillslopes and zero-order drainage basins to first order to n^{th} order basins, to subcontinental drainages. In these situations, the assumptions of HT are intrinsically present. For instance, first-order basins, in the aggregate, cannot influence third-order without translation through second-order basins (though individual stream segments may join higher-order channels). Similarly, molecular-scale atmospheric physics cannot influence local weather without translation through air parcels, which in turn cannot influence regional climate except via air masses.

Hierarchies may also be additive and clear in terms of their rank order, but less so in terms of causal chains because the physical boundaries are not always clear (e.g. individuals, populations, communities, ecosystems and ecological landscapes). Some hierarchies are imposed by nested scales or resolutions of maps or mapping programs or pixel sizes. Their conformity to HT is contingent on the extent to which the resolutions correspond to characteristic nested scales of processes or functional relationships, which is often uncertain, as resolutions are often dictated by technology or practical measurement constraints. In still other cases, hierarchical levels are based on conceptual models with possibly fuzzy or arbitrary boundaries, but which are widely used and generally agreed upon within a research community (e.g. the widely used paedological hierarchy originally presented by Dijkerman 1974). These are valid from the HT perspective if the phenomena involved are functionally nested – in the case of the soil hierarchy, for instance, soil horizons cannot directly influence, or be influenced by, polypedons or higher levels independently of the intervening pedon level.

HT as a pedagogic or heuristic device is more common than analytical applications, but the latter exist. HT is a key tool for addressing scale linkage in a GIS context (Dikau 1990; Wu 1999; Wu and David 2002) and in geography more generally (Meentemeyer 1989; Pereira 2002). For example, Albrecht and Car (1999) developed a hierarchy-theory-based method for scale-sensitive GIS analysis. HT was applied to problems of choosing and integrating among scales in multiresolution remotely sensed data by Phinn et al. (2003). Bergkamp (1998) applied HT to analysis of runoff and infiltration interactions with vegetation and microtopography, and Yalcin (2008) showed that a hierarchical method produced more realistic results than alternative methods for mapping landslide susceptibility. HT has also been applied to cross-scale modelling of nutrient loading in hydrologic systems (Tran et al. 2013) and the detection of landscape boundaries in ecology (Yarrow and Salthe 2008). Fryirs et al. (2018) used hierarchical methods to link analytical and informational, communicative models of the Okavanga River delta. Haigh (1987) seems to have been first to propose HT as a tool for addressing scale linkage in geomorphology. HT in ecology goes back a bit further (see reviews by O'Neill et al. 1986; Pelosi, Goulard, and Balent 2010; Reuter et al. 2010).

For at least some hierarchies (functional and additive, for instance), the scale independence implied by HT is entirely consistent with intuition and experience. Many techniques exist for determining the characteristic spatial or temporal scales at various levels, allowing a quantitative determination of the relative scales, resolutions, or rates. Where the fundamental assumptions of HT hold true, scale independence between scales >2 levels apart exists.

Phillips (2016) was concerned with determining how relatedness varies with distance in a scale hierarchy, in a way analogous to spatial distance decay. Algebraic graph theory methods, which can be used more broadly to assess scale independence in geographical networks, were applied as discussed in the next section.

2.5. Algebraic graph theory

Geographical systems can be represented as networks and analysed using graph theory, with system components as the graph nodes and relationships between components as the links or graph edges. A graph's adjacency matrix is an $N \times N$ matrix, where N = the number of nodes. The entries are zero if the row and column components are connected, and non-zero otherwise. Here we will consider simple, unweighted, undirected graphs where entries are either zero, or 1 if the nodes are linked.

Two key measures from algebraic graph theory will be used. The spectral radius is the largest eigenvalue of the graph adjacency matrix **A**, which has **N** eigenvalues λ, such that $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_N$. Spectral radius ($\lambda_1$) is a standard measure of graph complexity, and is directly related to graph entropy (Mowshowitz and Detmer, 2012). The second is algebraic connectivity, which measures synchronization or synchronizability of the network. It is calculated from the Laplacian matrix **L** of **A**, where **L** = **D** – **A** and **D** is the degree matrix, where the diagonal represents the degree of each node and all other entries are zero. Eigenvalues of **L** are all positive except for the smallest, $\lambda(L)_N = 0$. Algebraic connectivity is given by the smallest nonzero eigenvalue, $\alpha = \lambda(L)_{N-1}$. Literal synchronization may or may not be applicable to scale hierarchies, but α also represents inferential synchronization, or the extent to which inferences or observations at one point in the network can be applied to other components (Phillips 2013). High algebraic connectivity and synchronization in a hierarchical network indicates relatively seamless scale linkage, and vice versa. These measures are discussed in more detail in texts on algebraic or spectral graph theory (e.g. Biggs 1994), and elsewhere in a geographical and geoscience context by Phillips (2013, 2016).

In a scale hierarchy consistent with HT, components at any level *i* will be connected to at least one other node at that level, and within a level full connectivity (every node connected to every other) may be approached or achieved. At least one component at *i* (and often several) will be linked to one or more nodes at *i* + 1 and *i* – *1*. None will be connected to nodes more than one level away.

There are no doubt other possibilities for using graph theory and network analysis to analyse scale independence, but thus far such analysis been based on a set of functional relationships within and between hierarchical scale levels – for example, links between the fundamental soil-forming factors at and between the levels of horizons, pedons, polypedons, etc. Phillips (2016) analysed five hierarchical-scale networks. One was a fully connected reference graph that does not conform to HT, and two were archetypal networks designed to represent structures likely to be found in geomorphological systems. Two were real world – a general model of the pedological hierarchy commonly used in pedology and soil geomorphology, and a specific graph model based on work on fluviokarst flow networks in central Kentucky. Complexity, as measured by spectral radius and entropy, increases with the number of hierarchical levels, at a linear or less-than linear rate. Synchronization, as measured by algebraic connectivity, declines at a greater-than-linear rate with the number of levels in the hierarchy. The α results indicate rapid decay of inferential synchronization as new levels are added or considered, and that synchronization and relatedness decreases to very low levels when there are >3 levels – consistent with hierarchy theory assumptions of independence of components more than one level apart.

Algebraic connectivity is bound by

$$4/ND \leq a \leq k(A) \tag{19}$$

D is the graph diameter, the maximum shortest path (number of links) between any two nodes, and *κ(A)* is vertex connectivity, the minimum number of nodes that could be removed to disconnect the graph. In a scale hierarchy, *D* is linearly dependent on the number of levels of the hierarchy, and *N* also grows with additional scale levels. The minimum α therefore must decrease rapidly as scales are added. In a hierarchical network, *κ (A)* depends on the minimum number of edges or links connecting adjacent levels. Maximum α is insensitive to the number of levels if the connectivity between adjacent levels is consistent, but minimum values are very sensitive. Algebraic connectivity in scale hierarchies in geomorphic systems is more sensitive than spectral radius or complexity measures to separations in scale.

In the next section, algebraic graph theory and the other approaches described in this section are applied to the same example.

3. Case study: response to climate change

A network model of ecological, soil, and hydrogeomorphic state factor interrelationships was analysed in Phillips (2019) to address potential complexity, resilience, and sensitivity in terrestrial ecosystem responses to climate change. Ecosystems can be delimited at a variety of spatial scales, and at any given scale of interest are influenced by biological, hydrological, pedological, and geomorphological processes and controls operating at both smaller and larger scales. Here, the state factor model is assessed with respect to scale independence.

The model is general and synthetic in that it represents a consensus understanding key factors involved in the establishment, evolution, and functioning of ecosystems, and the interrelationships among them. Classic sources include Jenny (1941, 1961), Stephens (1947), Major (1951), Perring (1958), and Matthews (1992), with an excellent synthesis by Huggett (1995). Examples of recent studies based on this general concept of key components and interactions are studies of changes in recently deglaciated terrains by Eichel et al. (2013); (2016)), Klaar et al. (2015), and Miller and Lane (2019).

State factor model components are substrate, propagules, climate, biotic establishment, hydrogeomorphic context, and soil. *Substrate* refers to the ground surface, or the parent material for pedogenesis. The supply of potentially reproducing individuals, seeds, rootstock, etc., available to colonize a site is indicated by *propagules*. *Climate* signifies factors such as moisture and temperature regimes and insolation that influence biological habitat and pedogenesis. Climate-related factors such as floods, aeolian processes, and geomorphic disturbances are included in *hydrogeomorphic context*. This also includes drainage, hydrologic status, and topography and also the erosional or depositional regime. *Biotic establishment* refers to colonization and persistence of organisms. Significant modification of the parent material by biological, chemical, and physical processes distinguishes *soil* from substrate. Interactions (graph links) shown in Figure 2 are summarized in Table 1.

Table 1, Figure 2 *here*

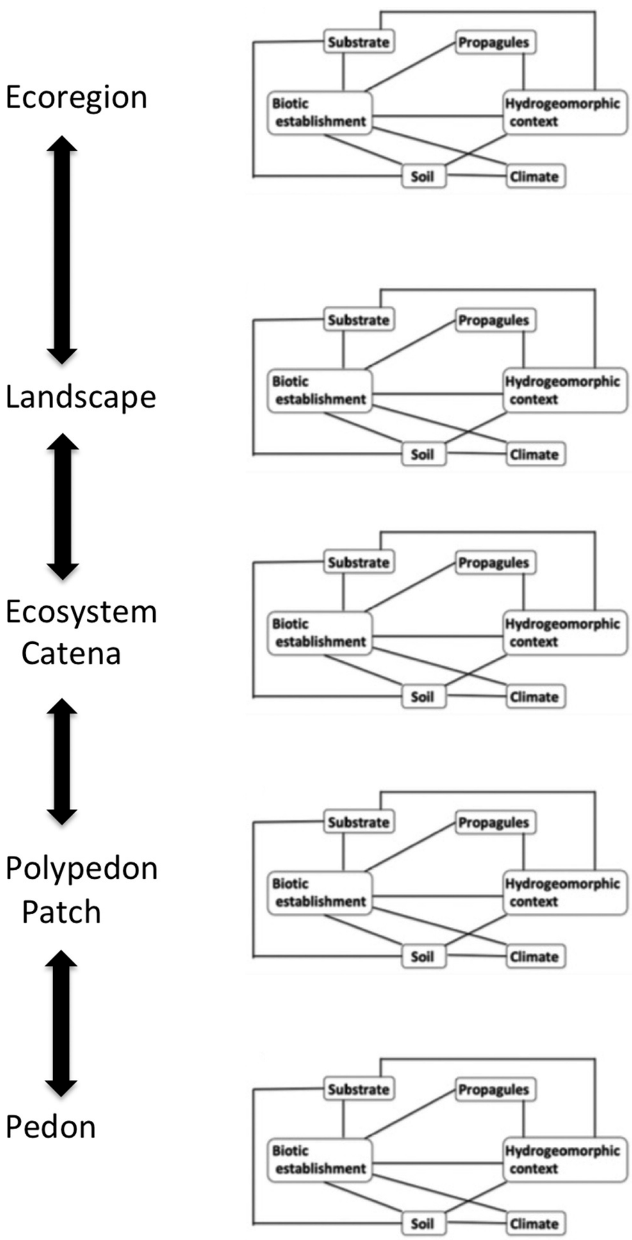

Figure 2. Hierarchical state factor model. Between-level links not shown for clarity.

Figure 2 shows the interrelationships among the state factors as a simple, unweighted, undirected graph, and a typical hierarchy of scales relevant to analysis of responses to climate change. The scale levels are generally accepted as functionally nested in physical geography, pedology, and ecology (O'Neill et al. 1986; Bouma et al. 1998; Wu 1999; Sauchyn 2001; Wu and David 2002; Schlummer et al. 2014).

Functional interactions among the state factors are considered to occur mainly within each hierarchical level. Links between levels occur factor-by-factor. For example, soil at the catena or ecosystem level is linked to the soil factor at the polypedon and landscape scales. Figure 3 shows the relationships between adjacent levels. In section 3 the arguments and methods described above (intuition, abstracted systems, hierarchy theory, K-entropy, graph theory) are applied to this case.

Table 1. Links between components of ecosystem state factor model.

Component	Links
Substrate	Parent material for soil formation; medium for biotic establishment; influences hydrologic responses & geomorphic processes. Substrate determined or influenced by hydrogeomorphic processes.
Propagules	Dispersal mechanisms for biotic establishment. Biota as sources for propagules; dispersal by hydrological & geomorphic processes.
Biotic establishment	Establishment is a function of propagule supply, medium (substrate), edaphic factors (soil, climate), and disturbance (hydrogeomorphology, climate). Establishment of organisms has reciprocal effects on all other factors.
Hydrogeomorphic context	Hydrologic responses, entrainment, transport & deposition processes influenced by substrate & soil properties & biota. Hydrogeomorphic processes important for dispersal & storage (e.g. seed banks) of propagules.
Soils	Soils are a function of parent material (substrate), climate, biota, topography & drainage (hydrogeomorphology), & soils have reciprocal effects on those components.
Climate	Climate[a] exerts influences through direct effects on soil & biota.

[a]Climate in this case represents moisture and temperature regimes and insolation. Climate-related disturbances are included in the hydrogeomorphic context component.

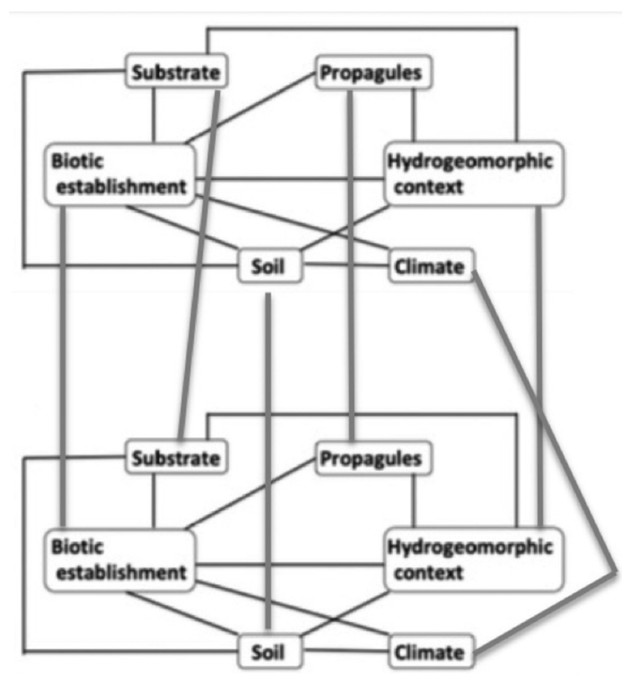

Figure 3. Black lines show links among state factors within hierarchical levels, and thicker grey lines links between levels.

3.1. Ecosystems and intuition

Responses to environmental change at any level of the hierarchy integrate the cumulative effects of lower levels. Using x to represent the hierarchical level of interest, $S(x)$ is the ecosystem state or condition at level x, and $F_i(x)$ represents the effects of processes or controls at level i manifest at x. Thus

$$S(x) = \sum_{i=1}^{q} F_i(x) \quad (20)$$

$$p[F_i(x)]\ \tilde{f}|x - i| \quad (21)$$

where $p[F_i(x)]$ is the probability of observing effects of level i at scale x, and the term at the right indicates that observation is a function of how far apart x, i are in the hierarchy. When $x = i$, $p[F_i(x)] = 1$. If the hierarchy conforms to HT,

$p[F_x(x)] = 1; 1 > p[F_{i-1}(x)], p[F_{i+1}(x)] > 0;$ and $p[F_i(x)] = 0$ otherwise.

So what is the probability of observing *direct* effects of an *individual* pedon (or plant or vegetation clump) at the ecosystem or catena scale, as opposed to effects mediated by the patch, gap, or polypedon scale? Or observing ecoregion-level effects directly at a level below the landscape, without mediation through the landscape scale? Experience indicates that it is very low, so the state factor hierarchy passes the intuition test for scale independence.

3.2. Abstracted systems and subgraphs

Passing the abstracted systems test requires that a dynamical system can be subdivided into subsystems of components that interact at particular scales, and of components connecting those scales. This is inherent in the structure of the state factor hierarchical models, where the subsystems shown in Figures 1 and 2, in any grouping of three or fewer levels, satisfy this criterion.

This test also requires that independent levels differ by at least two orders of magnitude. This is also the case for scales more than one level apart, as indicated by the characteristic length, area, time scales, and rates for processes at each hierarchical level (see, e.g. Pachepsky and Hill 2017).

3.3. K-entropy

The connectance entropy for the state factor graph can be calculated by

$$H_c = -S[(d_j/2m)ln(d_j/2m)] \quad (22)$$

Table 2. Connectance entropy (H_c) for the state factor model.

Level	N_i	H_c
1	6	1.735
2	12	2.452
3	18	2.857
4	24	3.146
5	30	3.336

where d_j is the degree of the j^{th}node, with connectance entropy as a surrogate for the *K*-entropy (Phillips 2002). Results are shown in Table 2.

With reference to eq. (18) and taking H_r as the entropy at level 5, and entropy at level 4 as representing $\Sigma[N_l H_l]$, we obtain $N_g H_g\ q \geq 0.190$. If an additional smaller level was added below the pedon scale, with its entropy the same as that for $i = 1$, then a qualitative causal shift would occur if N_l for that new level is >9.132 – less than an order of magnitude, and quite likely in the state factor hierarchy. Thus, while a direct quantitative test is not possible with the generalized model considered here the state factor hierarchy quite plausibly also passes the entropy test for scale independence.

3.4. Hierarchical structure, synchronization and scale

If the state factor hierarchy meets the criteria for hierarchy theory, then scale independence exists between non-adjacent levels. The state factor model is both spatially and functionally nested, and is also consistent with hierarchy theory as applied in pedology, ecology, hydrology, and related fields (Pachepsky and Hill 2017). The intuitive arguments in section 3.1 also support the consistency with HT.

Algebraic graph theory tests can also be brought to bear. Spectral radius and algebraic connectivity were calculated for the state factor system considering one to five levels. Because the within- and between-level graph structures are identical throughout the hierarchy, in this case it does not matter which levels are indicated (this is not always the case; c.f. Phillips 2016). Algebraic connectivity decreases rapidly for more than two levels, while spectral radius increases more slowly. Figure 4 shows λ_1 and α relative to the values for a single level, and to the maximum possible for a graph of a given N. For both λ_1 and α, maximum possible values are equal to $N - 1$ and would apply to fully connected graphs where every component is connected to every other. Given the number of state factors n, $N = n\ q$, where q is the number of hierarchical levels. The algebraic connectivity results are consistent with scale independence, given the rapid decline with distance in the hierarchy.

Table 3, Figure 4 *here*

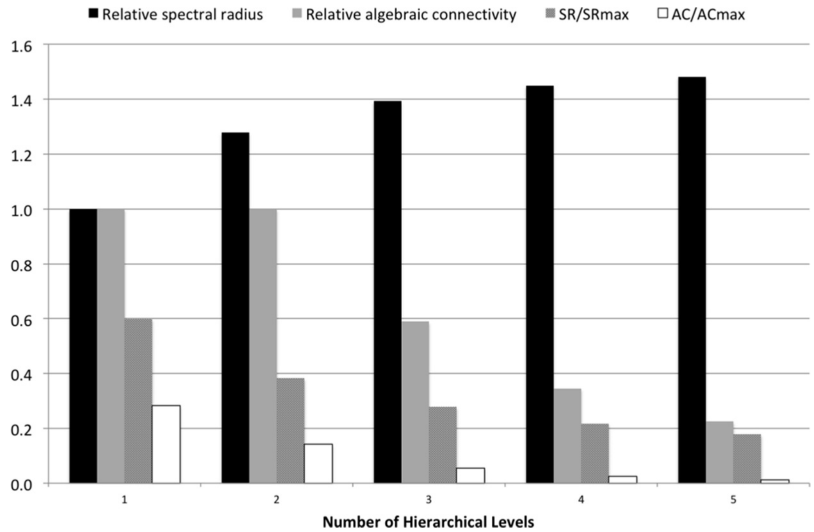

Figure 4. Spectral radius and algebraic connectivity relative to the values for a single level, and compared to the maximum possible for a graph of a given *N*.

Table 3. Complexity (spectral radius) and synchronization (algebraic connectivity) measures for state factor model with up to five hierarchical levels.

Levels	Spectral radius	Algebraic connectivity
1	3.593	1.697
2	4.593	1.697
3	5.007	1.000

4. Discussion

The principle of distance decay in geography, part of Tobler's (1970) First Law of Geography, has an analogue with respect to scale. That is, the greater the difference in spatial or temporal scale, the less likely things are to be directly interdependent. Unlike distance decay, however, scale independence can be explicitly stated as a law: processes and controls that operate at sufficiently different spatial and/or temporal scales are independent of each other with respect to their effects on system function and evolution. Given the broad range of scales encountered in geography and geosciences, from molecular to planetary and from instantaneous to billions of years, scale independence is highly relevant – though, of course, many specific problems can be defined and analysed based on only a portion of these vast-scale ranges.

The Law of Scale Independence is consistent with intuition and experience, and also holds for any hierarchical or nested arrangement of scales if the fundamental assumptions of hierarchy theory hold. In both instances, the scale independence can be tested quantitatively for specific cases. Algebraic graph theory also shows decreasing scale dependence with distance in a scale hierarchy, and the K-entropy analysis shows that scale independence is inevitable if the difference in resolutions in great enough. The abstracted eigenvalues theorem shows that scale independence exists with respect to effects on system dynamics. Together, these lines of argument converge to support the LSI.

The LSI signifies that we should not be surprised when different explanatory factors come into play at different scales. The law suggests a research strategy of identifying and focusing on the most important or interesting scale levels, rather than attempting to identify the smallest or largest scale levels and work top-down or bottom-up from there.

The answer to every question, it is sometimes said in the Earth and environmental sciences, is *it depends on the scale*. There is in this truism an element of observational or epistemological scale contingency, in that what can or cannot be observed is frequently conditional on scale in the form of resolution. There also exists phenomenological or ontological scale contingency, in that the dominant controls over Earth phenomena vary with scale, and processes operate, and patterns are manifested, over different time and space scales. The LSI solidifies this commonsense notion as inevitable given the range of scales encountered. Scale contingency is an innate, unavoidable aspect of Earth system evolution.

5. Conclusions

Processes and phenomena that operate at sufficiently different spatial or temporal scales are independent of each other with respect to their effects on system dynamics and behaviour. This is the *Law of Scale Independence*. The broad range of temporal and spatial scales encountered in geosciences and geography make it certain that the 'sufficiently different' standard is frequently met.

The LSI is supported by common sense, scientific and professional experience (and authority) and intuition. This intuition can be assessed and evaluated quantitatively for specific cases, if not formally tested, using ratios of characteristic lengths, areas, volumes, rates, and durations of phenomena involved (see examples in Phillips 1995, 1997). For the case of geographical systems represented as (nonlinear) dynamical systems, scale independence can also be demonstrated using the product theorem for abstracted eigenvalues. Entropy analysis also supports the LSI. Kolmogorov entropy of multiple-scale processes and controls varies with scale and can result in qualitative causal shifts when scale is expanded sufficiently in either (broader or more detailed) direction.

The basic assumptions of *Hierarchy Theory*, often applied in geography, ecology, and geosciences, ensure that non-adjacent hierarchical scale levels are independent. When hierarchies are defined, as they often are, on spatially and functionally nested levels, these assumptions are readily met. Algebraic graph theory, applied to hierarchies represented as networks or graphs, shows a rapid decay in inferential synchronization with differences in hierarchical position, consistent with the LSI.

A previously used state factor model for assessing ecosystem response to climate change was evaluated here on five hierarchical levels, from pedon to ecoregion. The arguments above – intuition, abstracted systems, K-entropy, hierarchy theory, and algebraic graph theory – were applied as tests of scale independence. All tests were passed, supporting the relevance of the LSI for a broad range of problems in physical geography and geosciences.

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Disclosure statement

No potential conflict of interest was reported by the author(s).
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