MUSICAL ALGORITHMS AND FUNDAMENTAL STRUCTURES

by

Eric Meehan

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Director of Thesis: Dr. Mario Rey

Major Department: Music Theory, Composition & Musicology

By writing algorithms to complete counterpoint exercises in the style of Johann Joseph Fux's *Gradus ad Parnassum,* a hierarchy of preferences between the given rules can be determined by testing different prioritizations of those rules. While conducting the study, it was found that strict adherence to any prioritization of rules led to deterministic or near-deterministic results, indicating the possibility of a 'fundamental structure of counterpoint' similar to the fundamental structures described by Heinrich Schenker. The study was conducted using four iterations of an algorithm capable of completing counterpoint exercises in the first species. Subsequently, implications of the findings are discussed along with recommendations for future iterations of algorithmic design.

Source code is available through GitHub:

https://github.com/eric-meehan/MusicalAlgorithmsAndFundamentalStructures

MUSICAL ALGORITHMS AND FUNDAMENTAL STRUCTURES

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Eric Meehan

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by

Eric Meehan

APPROVED BY:

DIRECTOR OF
THESIS: ____
Mario Rey, PhD

COMMITTEE MEMBER: ____
Thomas Huener, PhD

COMMITTEE MEMBER: ____
Elliot Frank, PhD

CHAIR OF THE DEPARTMENT
OF MUSIC THEORY: ____
Thomas Huener, PhD

DEAN OF THE
GRADUATE SCHOOL: ____
Paul J. Gemperline, PhD

TABLE OF CONTENTS

LIST OF FIGURES v

CHAPTER 1: INTRODUCTION 1

CHAPTER 2: REVIEW OF THE LITERATURE 3

  Pedagogical Approaches to Species Counterpoint 3

  Gradus Ad Parnassum 5

CHAPTER 3: ALGORITHMIC COMPOSITION 8

  Methodology 8

  First Iteration 13

  Second Iteration 16

  Third Iteration 20

  Fourth Iteration 22

  Results of the First Four Iterations 24

CHAPTER 4: IMPLICATIONS 27

  Heinrich Schenker and Felix Salzer 27

  Sigmund Freud 29

  Fifth Iteration 31

CHAPTER 5: CONCLUSION 36

REFERENCES 37

LIST OF FIGURES

1. First Iteration Example A 14
    a. Scatter Plot 14
    b. Western Notation 14
2. First Iteration Example B 14
3. First Iteration Example C 15
4. Second Iteration Example A 17
5. Second Iteration Example B 17
6. Second Iteration Example C 18
7. Second Iteration Example D 18
8. Second Iteration Example E 19
9. Third Iteration Example A 21
10. Third Iteration Example B 21
11. Fourth Iteration Example A 23
12. Fourth Iteration Example B 24

I. INTRODUCTION

A sufficiently precise instruction manual for creating a peanut butter and jelly sandwich should lead whomever follows its directives to said sandwich every time—even if that person has no prior experience with sandwiches. This is the principle from which the present thesis began. If we as music theorists aspire to making demonstrable claims that rise to the rigor of scientific scrutiny, we must reach this level of specificity in the claims themselves. Through testing the specificity of Johann Joseph Fux's counterpoint treatise, *Gradus ad Parnassum*, however, a wholly different conclusion was drawn—one that led to questions about the purpose of counterpoint exercises and deeper conclusions about its significance previously suggested by Heinrich Schenker and Felix Salzer.

The experiment was designed to isolate each variable of Fux's treatise in order to discover a true hierarchy of preferences in writing counterpoint. *Gradus ad Parnassum* discusses how the exercise prefers contrary and oblique motion over similar motion, and imperfect consonances over perfect ones; however, it does not give any indication of which of these principles is the most fundamental. Given the option, should an imperfect consonance in parallel motion be chosen over a perfect consonance in contrary motion? Fux's treatise, in this regard, lacks specificity. It would be simple enough to take each of the priorities discussed in *Gradus ad Parnassum*, arrange them into every possible order of importance, and then complete counterpoint exercises following that hierarchy; however, a well designed experiment eliminates the possibility of bias on the part of those conducting it, and anyone familiar enough with counterpoint to execute Fux's instructions will consequently possess certain musical preferences that may attenuate complete adherence to that hierarchy. To eliminate human bias, the experiment would need to be conducted algorithmically. A computer does exactly what it is instructed, no more or less, every time it is executed; moreover, it has no understanding of music and therefore cannot be biased in its execution of the instructions. While it may seem strange to conduct experiments on counterpoint treatises through algorithms, the process, in fact, suits the medium. As scholars such as David Cope have discussed, counterpoint treatises are already algorithms in and of themselves:

> "Johann Joseph Fux established another algorithmic-like process by describing many of the basic contrapuntal techniques of tonal music. Although Fux was not the first to codify such rules, he was certainly the most notable and widely read…. [Fux's rules are] expressed simply but eloquently".[1]

For this reason, a precise execution of this algorithm will likely reveal facets of the instructions that may not be immediately apparent to readers.

[1] Cope, 7.

II. REVIEW OF THE LITERATURE

Pedagogical Approaches to Species Counterpoint

Gradus ad Parnassum has been historically well regarded as a manual for the composition of counterpoint. Education in counterpoint, whether included in a textbook, course, or private lesson, often begins with references to Fux, and follow the species based exercises laid out in his work. Alfred Mann writes about the seminal work:

> "Since its appearance in 1725, it has been used by and has directly influenced the work of many of the greatest composers. J. S. Bach held it in high esteem, Leopold Mozart trained his famous son from its pages, Haydn worked out every lesson with meticulous care, and Beethoven condensed it into an abstract for ready reference… and in more recent times Paul Hindemith said, 'Perhaps the craft of composition would really have fallen into decline if Fux's *Gradus* had not set up a standard'".[2]

Felix Salzer and Heinrich Schenker, also felt Fux's treatise was useful both in teaching counterpoint to students and understanding music at a more fundamental level, stating:

> "The method established by J. J. Fux in his *Gradus ad Parnassum* still offers the best solution to the basic problem of teaching counterpoint. I believe, however, as did Schenker, that Fux's approach can only be used with decisive modification. Though Fux thought of his *Gradus* as a method of composition based on the works of Palestrina, it should be considered merely as an elementary and preparatory discipline for one of the elements of tonal composition, apart from any problems of style".[3]

Salzer's comments reveal two problems for the present study: the abstract nature of species based counterpoint, and the prevalence of disagreement on the actual rules of counterpoint exercises. Most pedagogues agree on the artificiality of species counterpoint in general, as the strictness of the rules in these exercises do not necessarily reflect the techniques of free composition; moreover, the seeming lack of connection to free composition has raised questions as to what the rules of counterpoint actually are.

> "Considering its inherent artificiality, however, one may wonder how closely the guidelines of species counterpoint exercises should reflect the melodic and contrapuntal practices of actual compositions. It is surprising how dogmatic many instructors seem to be regarding this matter, as though the rules for species counterpoint exercises are sacrosanct. Anyone who examines various counterpoint manuals quickly realizes

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[2] Mann, Back Cover.

[3] Salzer, 53.

> that their writers frequently disagree with one another regarding the specifics of the rules. For instance, whereas Johann Joseph Fux does not allow the use of neighbor tones in either second or third species, others allow neighbor tones in third species but not in second species, and still others allow neighbor tones in both second and third species”.[4]

The question of which rule set to use, while an important question for curriculum design, was not the initial concern of this project—the original aim was, originally, to study Fux. The results of the experiment, however, necessitates that this question be returned to later, as it becomes a central point of concern in the findings. The point of artificiality in counterpoint exercises, on the other hand, questions the utility of such an experiment from the outset. If counterpoint exercises are not true representations of musical composition and the rules are arbitrarily altered from one instructor to another, what is the point of refining their instructions to such an extreme level of detail? Moreover, what is the purpose of completing counterpoint exercises at all? Kent Kennan argues that the utility of the exercise is in its ability to introduce musical decision making in a controlled environment:

> “The chief objective of counterpoint study, in the author’s view, is to awaken or sharpen in the student a feeling for the contrapuntal element that is present to some degree in virtually all music; to make him sensitive to the forces of opposition and agreement, tension and relaxation, direction, climax, and the like, that operate whenever two or more voices are sounded simultaneously”.[5]

Another, more general perspective, is provided by Markand Thakar:

> “Manuals of species counterpoint have been produced for two hundred fifty years, some to benefit primarily composers, others music historians, and still others theorists. Broadly stated, the aim of these manuals has been either to develop a facility with relations among tones, or to develop familiarity with a musical style”.[6]

As will be discussed in depth later, these are not the only view of the utility of counterpoint. Some theorists, such as Heinrich Schenker and Felix Salzer argue that these exercise reveals something more fundamental to tonal music itself, like a telescope revealing something more

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[4] Burstein, 8.

[5] Kennan, ix.

[6] Thakar, xiii.

fundamental about the nature of stars. Furthermore, examining a theory at such a high level of detail forces a thorough understanding of a subject. A theorist who does not hold themself to a standard of explaining their ideas clearly, accurately, and precisely is akin to an astrologer whose predictions are vague enough to satisfy anyone eager enough to believe them. For a science of music theory, however, predictions must be falsifiable, results must be replicated, and the title of theory should be held with the same regard as one of evolution by natural selection, gravity, and the germ theory of illness.

Gradus Ad Parnassum

Before discussing the rules Fux provides in his treatise, it will be useful to address the variety of perspectives that may be applied to *Gradus ad Parnassum* as a whole. Given the text's format as a dialogue and its function as a tool for instruction, it seems reasonable to assume that the instructions should be conceptualized sequentially—the algorithm for first species would be limited to the understanding developed in the first chapter. Another possibility is to view the work with the knowledge of the specific aim of the exercise. If the purpose of first species is to prepare the student to write second, and eventually fifth species, this goal could inform some of the decisions that will need to be made regarding the algorithm's design. Similarly, each exercise may be viewed by the algorithm either sequentially or tautologically. Though sequentiality seems implied by some of the discussions Fux's characters have about the exercises, it is never made explicit. It may be useful, at times, to start working at the ending or in the middle of an exercise. Since this experiment aims to tease out the optimal set of instructions for counterpoint exercises, all of these perspectives are variables that may be tested through different iterations of the algorithm.

Before introducing first species counterpoint, Fux begins with an introduction to the rules of motion between two voices:

> "First rule: From one perfect consonance to another perfect consonance one must proceed in contrary or oblique motion.
> Second rule: From a perfect consonance to an imperfect consonance one may proceed in any of the three motions.
> Third rule: From an imperfect consonance to a perfect consonance one must proceed in contrary or oblique motion.
> Fourth rule: From one imperfect consonance to another imperfect consonance one may proceed in any of the three motions".[7]

The discussion of first species counterpoint begins with three additional rules:

1. "The beginning should express perfection and the end relaxation. Since imperfect consonances specifically lack perfection, and cannot express relaxation, the beginning and end must be made up of perfect consonances…. It should be noticed that in the next to last bar there must be a major sixth if the cantus firmus is in the lower part; and a minor third if it is in the upper part".
2. "Each of [the cantus firmus] notes, now, should be set a suitable consonance in a voice above; and one should keep in mind the motions and rules which are explained in the conclusion of the foregoing book. Contrary and oblique motion should be employed as often as possible, since by their use we can more easily avoid mistakes. Greater care is needed in moving from one note to another in direct motion. Here, because there is more danger of making a mistake, even closer attention should be paid to the rules".
3. "… more imperfect than perfect consonances should be employed".[8]

These rules are described more succinctly by Robert Gauldin:

> "Fux in his *Gradus ad Parnassum* summarized the basic 'commandments' of melodic motion between consonant intervals: use only contrary or oblique motion in approaching a perfect consonance, and use any of the three motions in approaching an imperfect consonance".[9]

Fux also discusses the necessity to adhere to a single mode within the exercise by making a correction to his imaginary student's example of lower counterpoint:

> "… the counterpoint must be in the same mode as the cantus firmus…. Since, in this example, the cantus firmus is in D as the beginning and conclusion show, and you started with G, you have obviously forced the beginning out of the mode".[10]

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[7] Mann, 22.

[8] Mann, 28.

[9] Gauldin, 46.

[10] Mann, 31.

While this example refers specifically to the alteration the mode through a lower counterpoint, the point can be taken for the use of accidentals as well—an example in the dorian mode should not include C# in the middle of a passage. One further instruction is given regarding the prohibition of the tritone:

> "This *mi* against *fa* you have written in the progression from the sixth to the seventh bar by a skip of an augmented fourth or tritone which is hard to sing and sounds bad, for which reason it is forbidden in strict counterpoint".[11]

In order for these instructions to be executed by the algorithm, a system of musical notation will need to be developed that allows for pitch identification, intervallic calculations, and a concept of melodic direction.

[11] Mann, 35.

III. ALGORITHMIC COMPOSITION

Methodology

By using a machine to execute the instructions given by *Gradus ad Parnassum*, human bias can be eliminated from the experiment because the machine has no preconceived notion of what the resulting counterpoint exercise should look like; however, this benefit also creates a challenge, as the machine has no concept of music in general. Much like a student in a classroom, it must be taught the fundamentals before more complex algorithms can be created. Moreover, these fundamentals must be taught in a manner with which the computer is familiar.

From the very beginning of the experiment, the reliance of music theory on human intuition is made obvious. To humans, having multiple pitches named "A" is easy enough to understand. If you ask a musician if the pitch A equals A, they will be able to explain concepts such as octaves, harmonics, and frequencies to arrive at the conclusion that A sometimes equals A. To a computer, however, the same question has one answer—yes. To achieve the nuance of human understanding, more specificity is needed by the machine. By adding octave numbers, the question can be sharpened to achieve a more precise question and a more accurate result. Additionally, the point of naming pitches is not simply to designate their position on the grand staff, but also to reveal a relationship between them. The computer must know that F# and A are a minor third apart, while F natural and A span a major third. It must also know that A to F# is a major sixth, and that F#4 to A5 is a minor tenth. While one could delineate each of these relationships in code, it would be a tedious process. A more efficient way to achieve this level of detail is to translate our concept of pitches into a language more familiar to computers—one of numbers.

Each pitch between C2 and B5 can be associated with a number between zero and forty-seven using a data structure called a dictionary. With this structure, which resembles the pitch class nomenclature of twentieth-century atonal music, the algorithm will be able to easily take pitch names, perform calculations with them, and present results in a familiar format to humans. The most important aspect of this method is the ability of the machine to perform calculations. In this dictionary, the pitch F#4 is associated with the number thirty, and A5 is associated with forty-five. By subtracting the lower number from the higher one, the value of fifteen can be associated with a minor tenth. For further simplicity, this value can be reduced by twelve to give the value of three as a minor third. This arithmetic applies to every pitch on the staff. From any pitch in the dictionary, simply add three to its associated value to find the pitch a minor third above it.

With this system, melodies can be translated into a different data structure called an array. Unlike the dictionary, which holds an unordered set of value associations, an array stores an ordered set of individual values. *Gradus ad Parnassum* instructs the student to compose counterpoint lines above or below a given cantus firmus, each of which will need to be translated into this new system of notation for manipulation by the computer. The dorian mode's cantus firmi, for example, becomes [26, 29, 28, 26, 31, 29, 33, 31, 29, 28, 26] in this new system of notation. Given this melody, Fux instructs the student to use intervallic relationships to determine appropriate pitches for the contrapuntal line. With its ability to preform calculations established previously, this process will be much more efficient if interval classes are defined in a group. Using three more arrays, the numbers zero through eleven can be stored to represent perfect consonances, imperfect consonances, and dissonances. This allows the machine to calculate the difference between any two pitches, reduce the result by twelve until it is less than twelve (thus simplifying the results to be within an octave), and then check which group the result belongs to, thereby giving the algorithm a way to distinguish consonant from dissonant intervals.

This system of notation also allows for a simple method for determining melodic direction. Simply subtract the numeric value of the previous pitch from the numeric value of the current pitch, and the result of this calculation will be positive if the melody is ascending, negative if it is descending, and zero if the melody repeats a note. Perform this calculation on both the cantus firmus and the counterpoint, and the type of motion between each line can be ascertained as well.

Finally, the results of the algorithm's calculations will need to be displayed in a human readable format. It would be simple enough to print the array of selected numbers to the screen in the same format that it is stored on the computer—[26, 29, 28, 26, 31, 29, 33, 31, 29, 28, 26];[12] however, a more elegant illustration can be generated to resemble familiar notation. The language used for creating these algorithms, Python, includes a standard library named Matplotlib for generating graphs. With a scatterplot, the X axis can be used to represent measures while the Y axis can represent pitch (as presented in the figures below). On this chart, two data sets can be added to reflect the cantus and counterpoint melodies. After performing the

[12] This is not actually how the computer stores data. Even here, the bracketed array of numbers separated by commas is an abstraction from the binary code that actually hold the requisite values in subsequent memory locations.

necessary calculations, the arrays for each melody can be given to Matplotlib to create XY coordinates on this chart, and the resulting scatterplot conveys the generated melodies in a format that reflects them visually similar to western notation. For more rhythmically complex examples, some changes would need to be made to this format; however, this example is sufficient for the needs of this experiment.

The technique of translating musical notation into numerical notation is scalable to any level of detail that is needed for composition and analysis. Instrumentation, rhythm, timber, dynamics, etc. can all be defined using a system of numbers. This is evident from the existence and prominence of digitally stored music and midi files—which, after all, are simply collections of ones and zeros. All sounds are, fundamentally, frequencies of vibrations that can be represented as numbers and equations. All music, therefore, is fundamentally mathematical. This is not a popular view, as is evident by the criticisms leveled by authors such as Peter Schubert:

> "Aristoxenus, in the fourth century BCE, felt it was necessary to point out that the study of music is different from the study of other more abstract disciplines such as geometry. Perhaps the other music teachers in ancient Greece were too busy calculating ratios to be bothered teaching their students to hear or sing or play. I quote him because we need to be reminded of this again today, when music theory is often divorced from so-called skills classes and is treated a bit like geometry. Counterpoint in particular, once a staple of the theory curriculum, is sometimes taught as a non stylistic ('abstract') prelude to harmony, or has been reduced or even eliminated altogether".[13]

However, arguments such as these typically confuse practical claims for ontological ones. While he is correct from a pedagogical perspective, providing students only with ratios and diagrams will not lead to excellent musicians, it is still true to say that every aspect of music may be represented numerically and is thereby susceptible to calculation. In fact, the prospect of using

[13] Schubert, 13.

algorithms to compose music is not a modern artifact of a digital age. Some scholars, such as David Cope, view the compositional process of almost every musician as being in some way algorithmic:

> "Composers have utilized various types of algorithms in composing their music for centuries. Constraints of almost any kind require algorithmic solutions, whether such constraints are dictated by a composer or style. This seems especially true with algorithm defined as 'a set of rules for solving a problem in a finite number of steps.' Since we assume composers use finite numbers of steps and not infinite numbers of steps and that composing itself can be seen as a problem requiring solving, it seems natural to view the process of composing music as algorithmic, no matter who the composer is. Tonal part-writing, for example, which dictates rules limiting creativity, acts as an algorithm around which composers embroider their own stylistic choices. Fugues and other contrapuntal formalisms represent constraints and often require several limiting algorithms".[14]

As the creator of several musical algorithms, Cope's analysis of the algorithmic nature of music is well supported. His algorithm for emulating J. S. Bach, named Experiments in Musical Intelligence (E. M. I.), even competed against genuine works by Bach and University of Oregon professor Steve Larson in its ability to write original works in the style of Bach:

> "Larson suggested that a professional pianist play three pieces one after the other: one each by Bach, by EMI, and by Larson himself. The audience would then be asked to vote on who composed which piece. Larson was convinced that people would easily distinguish between soulful human compositions and the lifeless artifact of a machine. Cope accepted the challenge. On the appointed date hundreds of lecturers, students and music fans assembled in the University of Oregon's concert hall. At the end of the performance, a vote was taken. The result? The audience thought that EMI's piece was genuine Bach, that Bach's piece was composed by Larson, and that Larson's piece was produced by a computer".[15]

While computers may require a tedious level of specificity to understand simple concepts such as the grand staff, harmonic intervals, and melodic direction, their ability to perform mathematical calculations is super-human.

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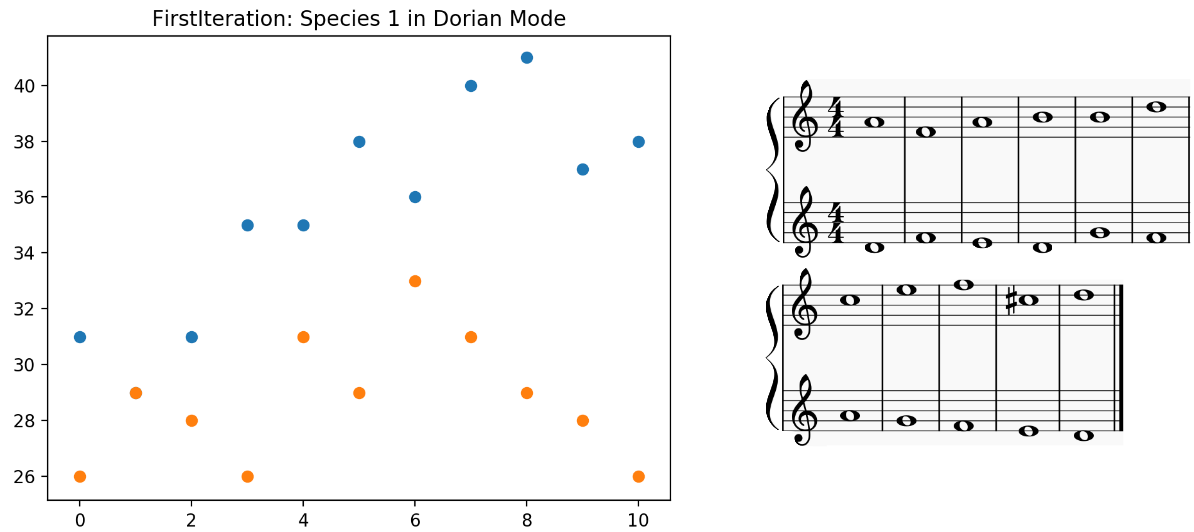
[14] Cope, 3.

[15] Harari, 329.

First Iteration

The first iteration of the counterpoint algorithm will attempt to adhere, as closely as possible, to the instructions given by J. J. Fux. It will begin by selecting a perfect consonance above the cantus firmus, then move sequentially through the melody looking for pitches that suit the requirements listed above. At this point, a hierarchy of preferences must be established, as the ordering of instructions will determine the pitch that is selected. To illustrate this, consider two hypothetical algorithms, one that sequences through imperfect and perfect consonances to find contrary motion, and another that sequences through contrary, oblique, and parallel motion to find imperfect consonances. From any given position in any given cantus firmus, there is almost certainly a perfect or an imperfect consonance in contrary motion to the bass; similarly, there is certainly an imperfect consonance that follows one of the three types of motion. As a result, the first algorithm will almost always employ contrary motion with a higher number of perfect consonances and the second will almost always utilize imperfect consonances with a high number of instances of parallel motion. Through software development, both options can eventually be explored; however, for this iteration, the first version will be employed. Once the algorithm reaches the second to last pitch of the cantus firmus, a major sixth will be selected, followed by an octave.[16]

[16] Note that the scatterplot titles are generated algorithmically using the name of the iteration as used in the code, hence the lack of spacing in "FirstIteration".



Figures 1a and 1b. First Iteration Example A (shown as a scatter plot and translated into Western notation)

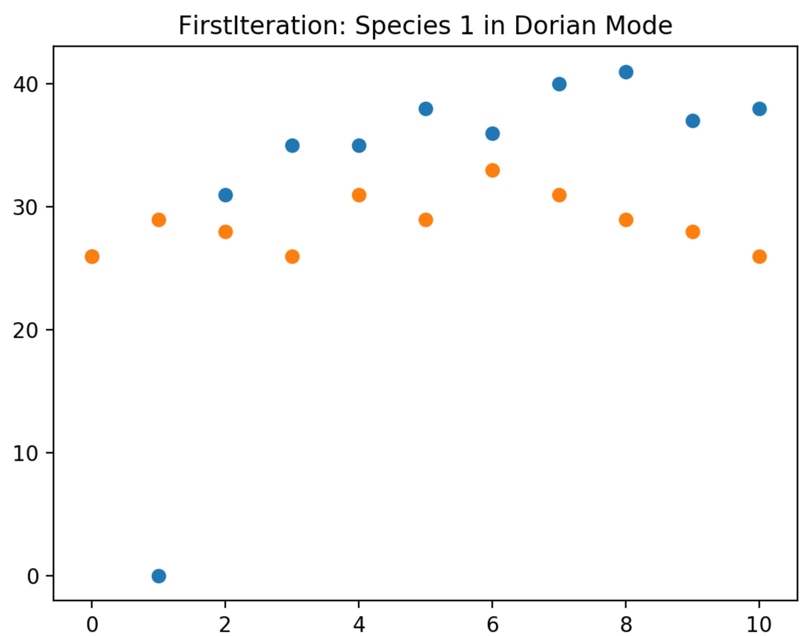


Figure 2. First Iteration Example B

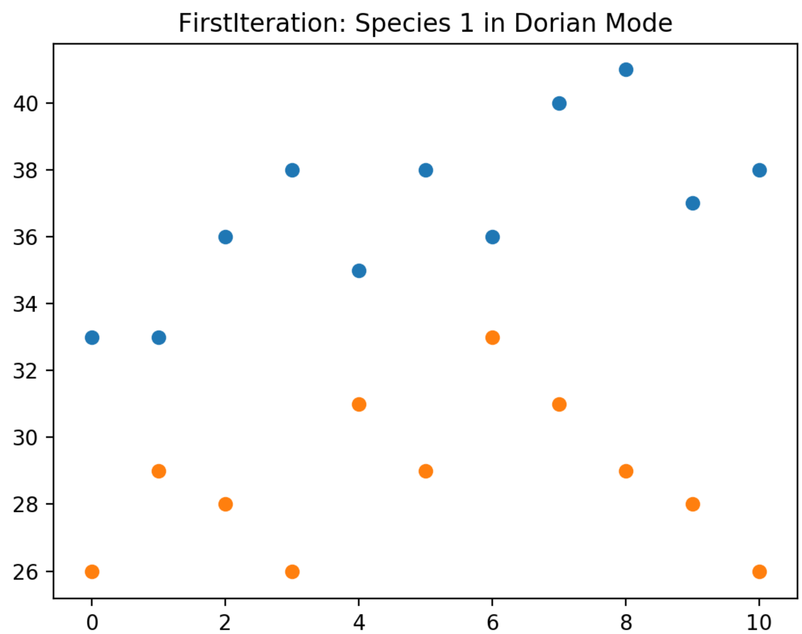


Figure 3. First Iteration Example C

Naturally, the results of this algorithm are not ideal or aesthetically satisfying examples of counterpoint. The line infrequently uses stepwise motion, no attempt is made to create a melodic climax, and the melody becomes fairly disjunct as it approaches the cadence. In the discussion of first species in *Gradus ad Parnassum*, however, none of these concepts are discussed. It could be argued that stepwise motion and a conjunct melodic line is implied when Fux discusses the prohibition of the tritone, his reasoning being that it is “difficult to sing;” however, this argument would be stronger if Fux stated that the melody should be easily sung, rather than a certain interval was particularly difficult to sing.

Another interesting feature of these results is that for each cantus firmus, there is only one result. While looking for pitches between the first and penultimate measures, the algorithm checks the list of imperfect and perfect consonances sequentially. Since it reads through this list the same way each time, it finds the same solution each time. To add some variability to the system, a random number generator can be employed to search these lists in an unpredictable order, hopefully resulting in more variability in the results. With the feature of having only one possible result, another discovery is made about this algorithm—there are instances where no solution is possible. To prevent errors, a line was added to the code that would select the pitch 0 if no suitable pitch could be found. One example of this occurring in practice was in the dorian mode when starting on a unison. Whenever and example begins on a unison in *Gradus ad Parnassum,* the counterpoint is always in the lower part and the lines frequently cross. Since this iteration only tests upper counterpoints and no protocols were included for allowing voices to cross, there was no possible solution to the problem. It would be of great benefit to students of counterpoint to have this point made explicit, as it will help avoid problems of this nature in their own practice.

Second Iteration

In this version of the algorithm, the first iteration has been copied exactly with a single alteration—the lists of imperfect and perfect consonances are searched randomly rather than sequentially. While this was done with the expectation that it would add a higher variability in counterpoint melodies, the results were surprisingly deterministic:

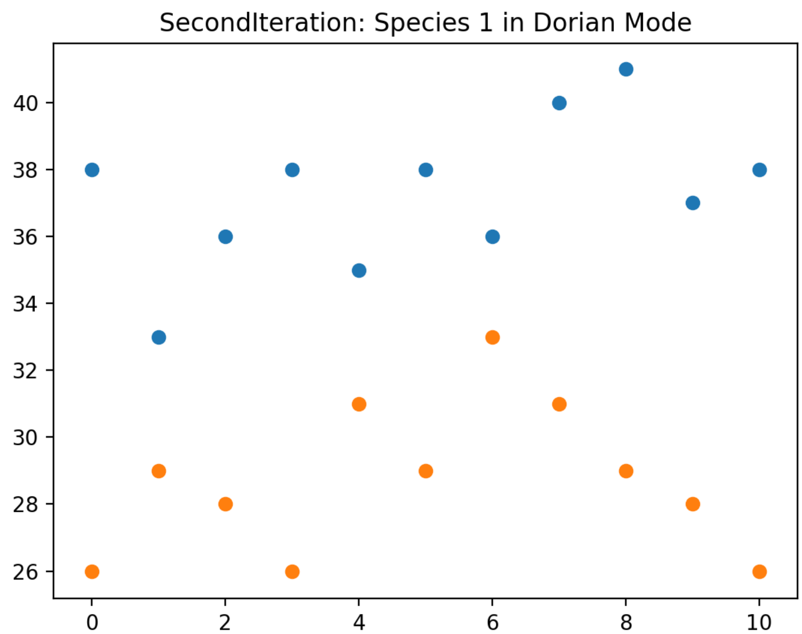


Figure 4. Second Iteration Example A

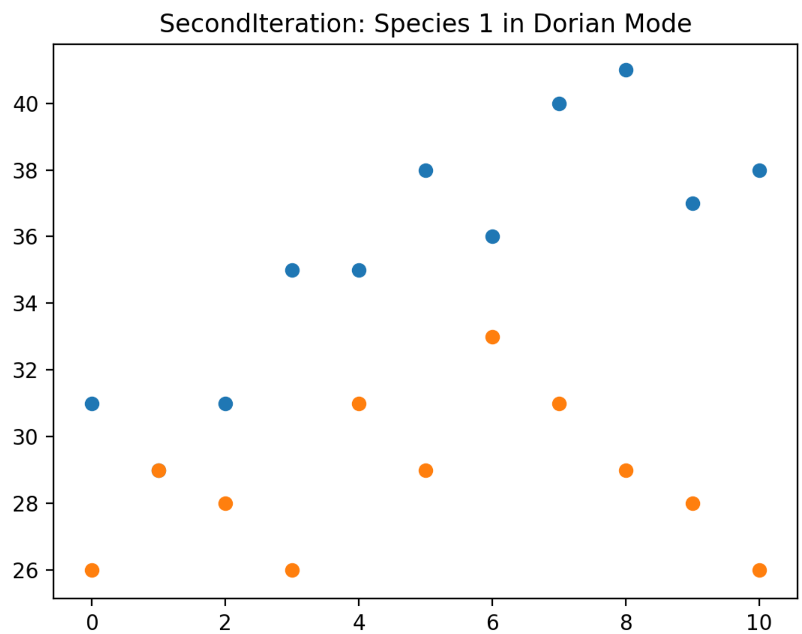


Figure 5. Second Iteration Example B

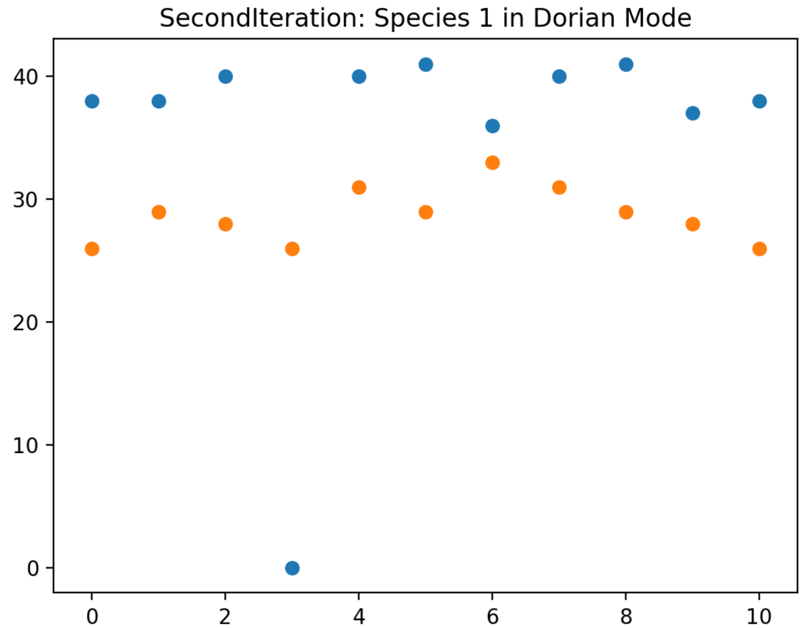


Figure 6. Second Iteration Example C

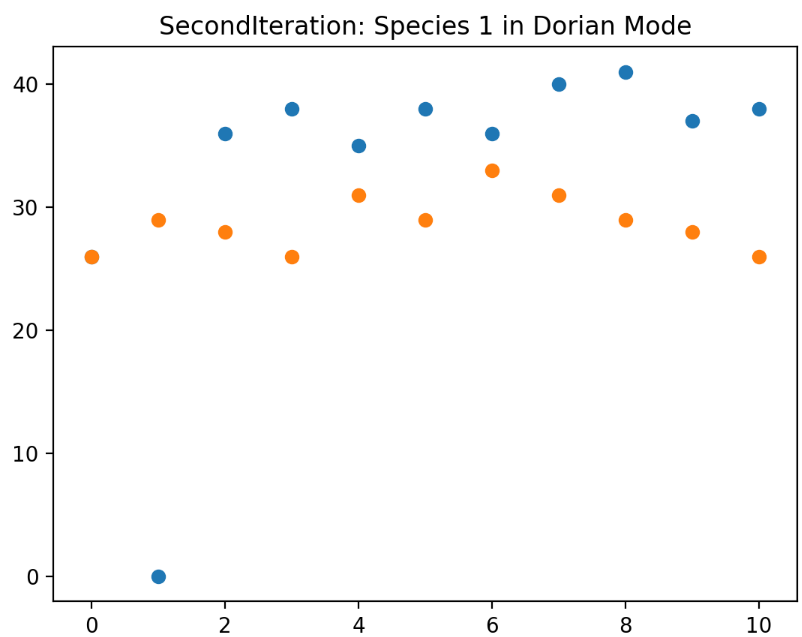


Figure 7. Second Iteration Example D

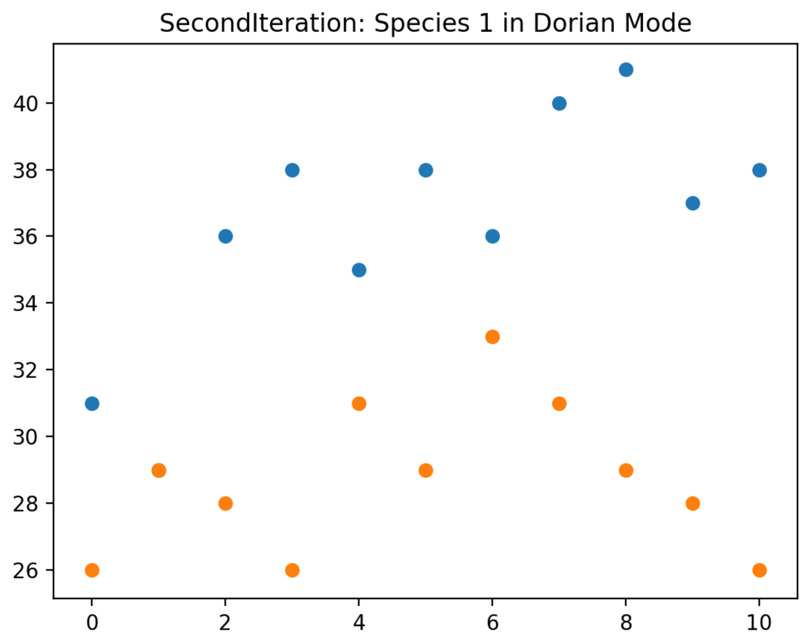


Figure 8. Second Iteration Example E

Some of the cantus firmi now have two results—differing by a single pitch. Not only has this alteration added virtually no variability to the system, but the new contrapuntal lines are exactly the same as those from the first iteration. Why hasn't randomness generated variability? It turns out that when preferences of contrary motion and intervallic relationships are ordered with sufficient specificity, there really is only one solution to the puzzle. Consider the first two pitches of the dorian cantus, D and F. A perfect consonance is required at the opening, so A may be chosen semi-arbitrarily. From there, the algorithm is first looking for an imperfect consonance that moves in contrary or oblique motion. Above any given pitch, there are four imperfect consonances: a minor third, major third, minor sixth, and major sixth. A minor third above F would be Ab and a minor sixth above F would be Db, neither of which are in the required mode. The algorithm is left with two options: oblique motion to A or similar motion to D. Since it has already been determined that contrary and oblique motion are preferred over similar motion, the only possible answer is A. Even with randomness added, there is one solution. This process remains true for virtually every pitch in the cantus firmus, as there will always only be two imperfect consonances within the mode, and at least one is likely to be in similar motion to the bass. Is this determinism the result of this algorithm, or does it reveal something more fundamental about the nature of counterpoint exercises? If the rules are reordered, would there be more options for pitch selection at each point in the melody? In the next version of the algorithm, the rules will be altered to test this question.

Third Iteration

Given that Fux has left out instructions on aspects of counterpoint widely seen as desirable, it will be useful to add them to the algorithm for two reasons: first, it will help create a more perfect instruction set for the creation of counterpoint, and second, it will allow for a test of variability in the results. For the third iteration, an emphasis will be placed on the use of stepwise motion. To accomplish this, the algorithm will still select the first pitch randomly from the set of perfect consonances and the last two according to the rules previously discussed. For the remaining pitches, however, it will first examine the pitch that is stepwise in the opposite direction of the cantus, then one that is stepwise in similar motion to the cantus. According to Fux's instructions, the former can be either an imperfect or a perfect consonance, while the latter must be an imperfect consonance. If neither pitch yields a suitable result, the previous algorithm will be utilized to fill in the gap.

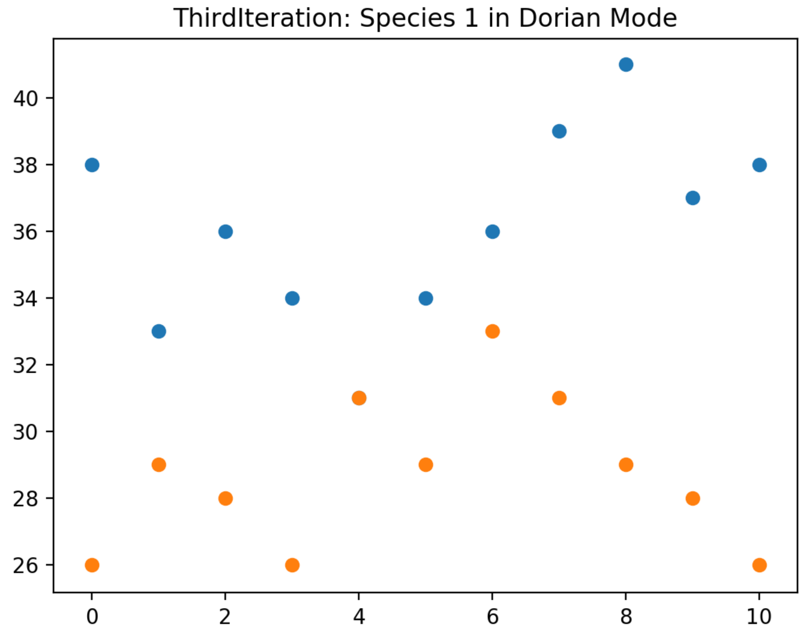


Figure 9. Third Iteration Example A

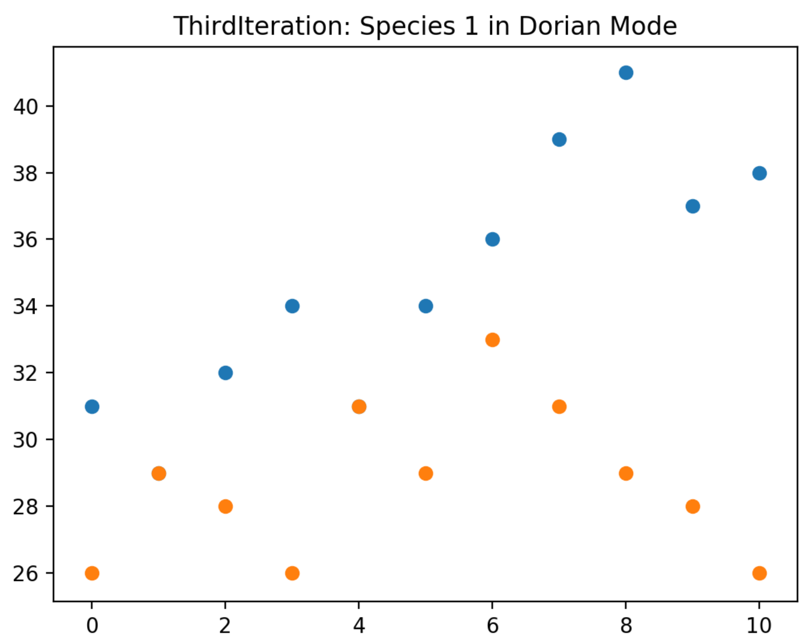


Figure 10. Third Iteration Example B

This iteration is much more conjunct than the previous two, though there are still some imperfections. As shown here by the algorithm, *Gradus ad Parnassum* would benefit from an addendum on the importance of stepwise motion and the necessity to search for it as the first option. As shown in iterations one and two, searching for anything else before stepwise motion will likely return a result, and if the practitioner accepts this result without examining the possibility of stepwise motion the melody is likely to be disjunct. Though contrary stepwise motion is still preferred over similar stepwise motion, stepwise motion in general must be preferred over contrary motion in general if conjunct melodies are desired. Future curriculums dealing with the composition of species based counterpoint would do well to make this point explicit.

Determinism remains a prevalent aspect of this iteration. This result, now, is somewhat unsurprising. The search for stepwise motion is performed sequentially, and the algorithm will take the first pitch it finds that suits the preferences it has been given. If none are found, it follows the same 'random' search performed in the second iteration, which was also deterministic. It seems that when the rules of counterpoint are made sufficiently specific, there is little to no flexibility in the system.

Fourth Iteration

The fourth iteration was a step backwards in terms of melodic independence; however, the goal of the experiment was not to make each iteration better than the last, but rather, to test different hierarchies of preference to assess which yielded the best results. This iteration is quite similar to that of the previous one with a bit more nuance in the selection of stepwise pitches.

Here, the algorithm prefers imperfect consonances over contrary motion. If the stepwise pitch in contrary motion is an imperfect consonance, the algorithm accepts it and moves on; however, if that pitch is a perfect consonance, it will first check the stepwise pitch in similar motion for an imperfect consonance. If that test also fails, the previous perfect consonance in contrary motion will then be accepted, thus ordering our preferences as stepwise motion, imperfect consonances, followed by contrary motion.

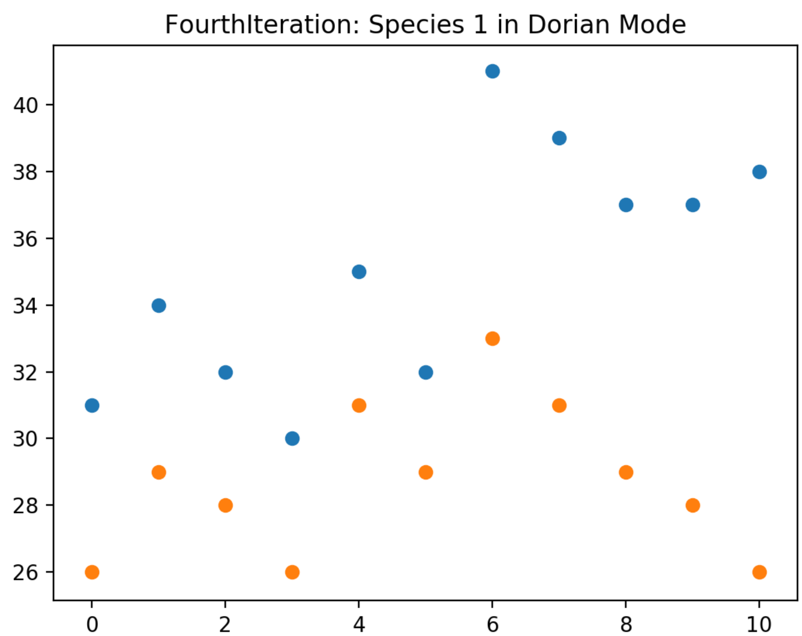


Figure 11. Fourth Iteration Example A

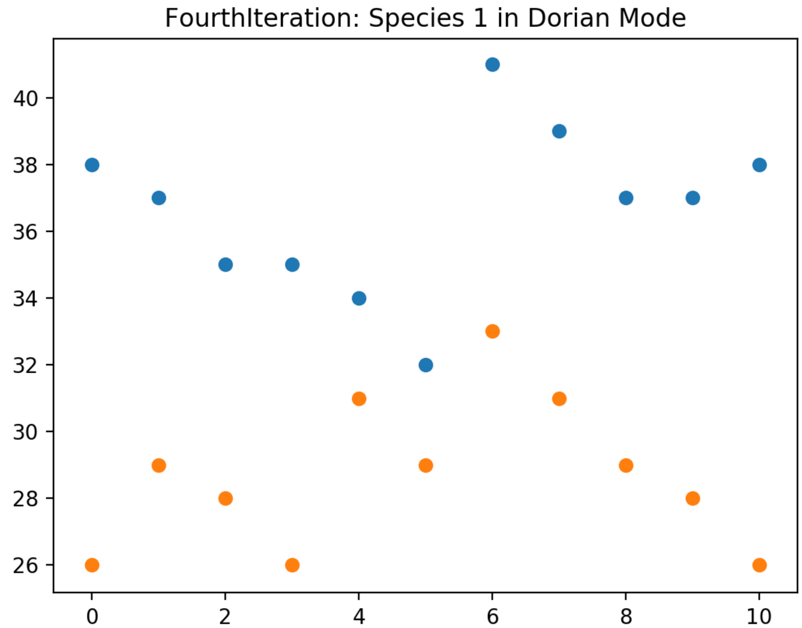


Figure 12. Fourth Iteration Example B

The results are, frankly, untenable. In almost every circumstance, the algorithm was able to find a suitable pitch in similar motion, thereby resulting in almost an exact copy of the cantus firmus. Given that the goal of counterpoint is to create independent lines, this cannot be the order of preferences in pitch selection. While not an improvement on the quality of calculated melodies, this iteration illustrates in a compelling manner that contrary motion should be preferred over imperfect consonances.

Results of the First Four Iterations

Programming the instructions given by Fux's *Gradus ad Parnassum* has thus far uncovered several potential areas of improvement for this treatise and counterpoint pedagogy in general: firstly, when beginning at the unison, the counterpoint should be in the lower part and the lines will generally need to cross. Secondly, stepwise motion in the counterpoint should take precedence over contrary motion, as the inverse will lead to highly disjunct melodies. Thirdly, contrary motion should take precedence over imperfect consonances in order to preserve the independence of the lines. Fourthly, and most interestingly, when properly prioritized and followed precisely, the results of counterpoint exercises are deterministic. This will be the most controversial finding of this study, as many educators will likely feel that the point of counterpoint exercises is missed if an optimal solution exists. Perhaps some will feel, as Poundie Burstein does, that counterpoint is an opportunity for students to explore composition in a controlled setting:

"Studying counterpoint in music theory classes helps to heighten students' awareness of melodic and contrapuntal effects that appear in a variety of musical settings, including those where the melodic and contrapuntal treatment is considerably freer than in species counterpoint".[17]

Other music educators may view the practice as an exercise in musical decision making, a perspective held by Peter Schubert:

> "Why teach counterpoint at all? Because counterpoint is more than combining nice lines, and it is much more than correct treatment of dissonance. It is a way of thinking in music that does not take for granted the static relationship of melody and bass, or a harmonic progression. Contrapuntal thinking means seeing, in two or more lines of melodic material, possibilities involving time shifts and transpositions, ways of ensuring variety of harmony and texture…. Knowing how composers thought about their music is essential to understanding it, and counterpoint provided many of the basic structures for all their music".[18]

Or even that counterpoint exercises are primarily useful in the context of developing skill in more performative areas of training, as is implied by Sarah Marlowe in her essay on the topic:

> "Including real musical excerpts during the earliest stages of species counterpoint study has important pedagogical benefits. Specifically, students will begin to make stronger connections between their abstract counterpoint exercises and real musical textures…. Hearing, performing, and analyzing real musical

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[17] Burstein, 9.

[18] Schubert, 24.

> examples will ultimately make the early stages of theory coursework—which can often feel tedious and frustrating for students—more fun and engaging”.[19]

All of these perspectives, however, are undermined by the findings of this experiment. Two conclusions can be drawn from the present result: either the hierarchy of values presented in *Gradus ad Parnassum* are to be applied with less rigidity, allowing for greater variance in results at the expense of contrary motion and imperfect consonances, or that the purpose of counterpoint exercises is not a creative endeavor, but rather one that reveals a more primordial structure of this style of music.

[19] Marlowe, 63.

IV. IMPLICATIONS

Heinrich Schenker and Felix Salzer

In the text *Structural Hearing*, Felix Salzer describes how the strictness of species based counterpoint reflects not a rigidity in musical composition itself, but the function of the exercise in introducing students to problems that are encountered in a tonal idiom and their proper solutions. He writes:

> "There is actually no contradiction between a composition by Beethoven and the theory of so-called strict counterpoint. There is rather a relationship comparable to that of structure and prolongation, or of basic idea and extension and development. Accordingly, rules and progressions of counterpoint should not be applied directly in composition. It is senseless to evaluate the rules of counterpoint to this of composition, senseless because counterpoint does not represent a study verging upon composition itself, but only the study of one of various elements forming a complex pattern. The average composition written on a tonal basis includes examples of harmony, counterpoint, form, rhythm, motif development, melodic prolongation, chromaticism, etc. If we wish to understand the essence of counterpoint, we must separate it from this complex pattern and study it in its purest form. For such a purpose, the Fux method is highly valuable; it is adaptable as a preparatory discipline even for compositions of far later periods. Problems of melodic contour in music of all styles may be approached through the medium of this contrapuntal discipline".[20]

As a student of Heinrich Schenker, Salzer's views on counterpoint are based on the analytical style of his teacher. Schenker's theory of tonal music gives a somewhat Platonic view of art, and the completion of a Schenkerian analysis on two different works often reveals highly reductive charts that may differ little from each other. He wrote of species based counterpoint in his treatise, *Kontrapunkt*:

> "Counterpoint must be separated from composition if the ideal and practical truth of both are to come into their own right…. The discipline of counterpoint is not meant to teach a specific style of composition, but to serve to lead the ear for the first time into the endless sphere of original problems in music…. Counterpoint must restrict itself on the basis of a modest exercise… to demonstrate the nature of the problems and their solutions, and should not attempt to be more than a preparation… for genuine composition".[21]

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[20] Salzer, 53.

[21] Schenker, 10.

A similar point is made by Poundie Burstein in his essay on species based counterpoint in a pedagogical setting:

> "The point here is not to advocate for one set of rules or another. Whatever rules are decided on, whether for species counterpoint or other music theory activities, it is important that the teacher keeps the main function of the exercises clearly in view. Teachers should not pretend, for instance, that in species counterpoint dissonant passing tones on the first beat of a measure should be avoided because they don't arise in real music: unless speaking about the specific style of the *stile Antico,* such claims clearly would be false. Studying counterpoint in music theory classes helps to heighten students' awareness of melodic and contrapuntal effects that appear in a variety of musical settings, including those where the melodic and contrapuntal treatment is considerably freer than in species counterpoint".[22]

While Burstein may not wish to advocate for one set of rules over another, the evidence gathered from the experiment described in this paper suggests that it is important that a consensus be reached on this matter.

The irony of drawing connections to Schenkerian theory in the present study is that Schenker's own theory of counterpoint draws the exact opposite conclusion than the one discussed previously:

> "Contrapuntal theory addresses the problems of voice leading by presenting them in the form of prescriptions and restrictions. That is, we experience one aspect of tonal activity in that counterpoint disallows one thing or another, while we experience a different aspect in that it prescribes a particular procedure. For the present, however, only this, which must be noted well: one should not be deceived by the large number of prohibitions; even though contrapuntal theory establishes many prohibitions, it is certain that far more is allowed. Even in the real world of written exercises, there is far more freedom than restriction! (Just as in life generally there is more freedom than restriction, and it is merely their own folly when people who stare and gape only at the prohibitions succumb to the illusion that the opposite is true)".[23]

Robert Gauldin describes how Schenker's perspective of species counterpoint was that the exercise was a prerequisite to analyzing fundamental structures in tonal music, not that the exercises themselves had such structures of their own:

> "Although some scholars, such as Knud Jepesesn, have continued to apply Fux's methodology to the study of sixteenth-century polyphony, others have used the basic principles to discover underlying contrapuntal voice-leading in general. Schenker advocated the acquisition of skills in harmony, thoroughbass, and species counterpoint as prerequisites to his method of reductive analysis".[24]

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[22] Burstein, 9.

[23] Schenker, 12.

[24] Gauldin, 291.

Why might Schenker have overlooked or rejected the possibility of fundamental structures applying to counterpoint itself? While such a question can only be addressed speculatively, the validity of his theory in general can be analyzed directly by examining his own publications. An investigation into how Schenker arrived at his conclusions does little to ease any methodological concerns.

Sigmund Freud

Sigmund Freud is known for starting the field of modern psychology with precocious views of the unconscious mind. He is also known for non-empirical theories such as the oedipus complex, castration anxiety, and dream analysis. One of the main criticisms of Freud's work is that his theories are so vague that they cannot be tested and proven wrong. Unfortunately, the same criticism applies to Schenker's writings. Consider the following passage from Schenker's *Counterpoint*:

> "Regardless of all the freedom of free composition, even there the first principle of the theory of counterpoint—'In the beginning is consonance!'—has practical significance: even in free composition, that which, as dissonance, cannot and may not be substantiated, must be placed upon the foundation of consonance. If only the composers of today could at last understand how utopian it is to believe that the nature of our senses could ever grant the dissonance an equal birthright alongside consonance! The two of them, consonance and dissonance, cannot have the same role; that is assured by the basic law of nature in general: never form a thing twice in the same way".[25]

His fundamental claim in this passage, that dissonance must be supported by consonance, is completely unsubstantiated. Schenker claims this is based on 'the basic law of nature' without providing any explanation to indicate that this 'law' relates to musical composition, is relevant in the study of counterpoint, or even exists. Even if he is correct about the dependance of

[25] Schenker, 112.

dissonance on consonance, his defense of this claim is, frankly, bizarre. This issue is abundant in Schenker's writings:

> "The absolute character of the world of tone, as one discovers it for the first time in the study of counterpoint, means that music is emancipated from every external obligation, whether it be words, the stage, or the narrative aspect of any kind of program. The self-referential nature of tones obliges the composer to adapt himself to their inner life, and to relegate to a lower place any purpose that may have been associated with music. This means that the consideration of a secondary purpose must never become the principle aim; the tones themselves, through bad effects, would have to protest against any such secondary purpose".[26]

Even if Schenker had an accurate insight into a fundamental pattern in music, it was mixed in with these fantastical arguments, making it difficult to discern what is a well considered position and what is not even wrong. Given the questionable methodology demonstrated by much of his writing, it seems that when Schenker is right it is not due to a rigorous standard of testing, but more by the fortune of taking an educated guess. This causes serious problems for the theory itself, as when it comes to ideas that lack experimental evidence, the valid ones are questionable by association with the invalid. Consider the following passage from Schenker's *Knotrapunkt*:

> "Free composition can dispense with an actual distinct extension in the time of the organizing tone and point only ideal tones that can be expected to bear the burden of dissonances. Yet these ideal tones certainly are so completely present in our consciousness that they can, in this sense, again be described as actual. First and foremost in free composition it is the scale degrees that have their own secret law of progression, and precisely our intuitive familiarity with that law of progression makes plausible the assumption of those ideal tones that lie outside the realm of actual voice leading".[27]

Here, Schenker makes an ontological claim regarding the existence of 'ideal tones' based a neurological claim about human perception. Readers can be almost certain that Heinrich Schenker's background in neuroscience does not qualify him to make such a claim. How, then, can the claim regarding 'ideal tones' be taken seriously given its proximity to such an unjustified speculation? Fundamentally, Schenker's writings are illustrative of the inherent liability in writing theories that lack experimental methodologies—it is impossible to tell fact from fiction.

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[26] Schenker, 15.

[27] Schenker, 112.

Eventually, someone will have to conduct an experiment to verify what a given theorist is arguing, and failing to do so within the context of the original argument is simply creating more work for future scholars. After Sigmund Freud wrote his theories, psychologists had to spend years going back over his work experimentally, verifying some claims while falsifying most. The same, unfortunately, must be done for Schenkerian theory. While the present study may indicate conclusions similar to those found in Schenker's work, it would be inadvisable to attempt to make experimental results fit the theory, rather than allow the theory to arise from experimental results.

Fifth Iteration

Having discussed several possible algorithms that prioritize different values presented in Fux's *Gradus ad Parnassum*, and having explored the implications of the results, it should be noted that there is still much room for improvement before an optimal algorithm can be designed. Earlier in this study, it was mentioned that there exists a disagreement among scholars regarding what the specific rules of species based counterpoint should be. Authors such as Poundie Burstein speak with disdain of the "dogmatism" with which some scholars adhere to one set of rules over another; however, given the deterministic results of this study, it seems important that the rules that define species counterpoint be considered carefully. To this end, a future iteration of this algorithm would benefit from a comprehensive survey of counterpoint treatises. From this, the variables that are to be tested algorithmically can be narrowed, as the areas where theorists are largely in agreement can be taken as a base line instruction set and the remaining differences can be programmed individually.

Though it would be an exhaustive process, a comprehensive survey would also aid in the creation of a more complete algorithm. By drawing from a single source, gaps in instruction are likely to exist in the code. It was discovered in the present study that stepwise motion must be given a high priority when writing species exercises. While this instruction is absent from *Gradus ad* Parnassum, the eighteenth-century theorist, Luigi Cherubini, makes this heirarchy explicit: "All movement should be diatonic or natural in regard to melody; and conjunct movement better suits strict counterpoint than disjunct movement".[28] Even when there is overlap between sources, one may often provide a clearer or more thorough explanation of a given rule. For example, Arthur Merritt's discussion of species counterpoint describes the perfect fourth as a dissonance just as Fux does, his reasoning behind this rule, however, is much more enlightening: "The perfect fourth is very common as a melodic interval, but as a harmonic interval it is always considered a dissonance and is treated like all other dissonances".[29] More often than not, there will be significant overlap between sources, as the following example from Cherubini illustrates:

> "Except in the first bar and the last, imperfect concords should be employed in preference to perfect ones. The object of this rule is to produce harmony by means of imperfect concords, which are more acceptable than others".[30]

The differences, however, are where algorithmic experiments become useful. Cherubini makes an exception to Fux's prohibition of direct motion to perfect consonances: "Passing to a perfect concord by direct movement is prohibited, except when one to the two moves a semitone. This exception is tolerated",[31] and there are unquestionably many more idiosyncrasies that can be

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[28] Cherubini, 9.

[29] Merritt, 126.

[30] Cherubini, 11.

[31] Cherubini, 9.

discovered between treatises. The next stage of this study is to find the fundamental set of rules common to all of these treatises, leaving only their distinctions to be tested.

When reconciling the differences between treatises, the algorithm may need to change drastically between iterations. Fux, Cherubini, and Salzer all discuss the process of completing a counterpoint exercise in a more or less linear fashion—the student starts at the beginning and moves to the end. The algorithms used in this study have followed a similar pattern; however, it is possible that an algorithm may need to approach the exercise differently depending on the instructions being implemented. Consider, for example, this excerpt from Kent Kennan's counterpoint textbook:

> "In most tonal melodies certain notes form an underlying skeletal structure, a kind of line within a line… Not all 'basic shapes' are so apparent to the ear…. Some are obscured by digressions, by the implying of two or more voices within a line, or by the transfer of a musical idea to another voice. Still, it seems safe to say that the most successful melodies are this in which the underlying sense of direction is clear".[32]

With the addition of a 'skeletal structure,' an algorithm made for this instruction may need to address sections of the melody out of order. Such an algorithm may even be irreconcilable with a linear methodology. The results of each, though, could be compared to test the strengths of one approach over another.

Coding each iteration of this algorithm is a painstaking process. For every possible ordering of contrapuntal preferences, the logic of the algorithm must be worked out and the code written, debugged, and tested. For this project, the shortest algorithm required one-hundred-sixty-four lines of code. While there is likely a more efficient way to achieve the same logic, making the algorithm more efficient simply adds to the amount of work that needs to be done for each step; moreover, this project only tested first species counterpoint between two voices. As

[32] Kennan, 4.

more voices are added to the texture and the number of pitches per measure increases, these algorithms will only grow increasingly complex, and accounting for differences between respective treatises makes this a daunting task. To address this issue, the experiment could be reengineered to deduce the instruction set by examining examples of completed exercises. Using machine learning, the amassed data set of exercises from a variety of sources can be algorithmically analyzed to generate a model of the rules governing their creation. Highly publicized today, machine learning algorithms are widely known for two properties: their uncanny ability to outperform human competitors at specific tasks, and their inability to explain how they accomplished this feat. To illustrate this, consider the victory of Google's AlphaGo software over the South Korean Go champion, Lee Sedol, in 2016:

> "AlphaGo trounced Lee 4-1 by employing unorthodox moves and original strategies that stunned the experts. Whereas prior to the match most professional Go players were certain that Lee would win, after analyzing AlphaGo's moves most concluded that the game was up and that humans no longer had any hope of beating AlphaGo and its progeny".[33]

AlphaGo was not given explicit instructions on how to play Go. Rather, it learned by playing millions of rounds of the game against itself, analyzing the results of each as a data set. If a similar procedure were applied to examples of species counterpoint, similar results should be expected—a model of counterpoint so precise that its execution would either be indistinguishable from or more perfect than human compositions. Much like the 2016 Go match, though, experts in the field may find the results inexplicable, and the algorithm would be completely incapable of explaining how its model was functioning. In order to make this experiment useful, the explainability problem would need to be solved. This is, in many ways, perhaps more difficult the the previously discussed methodology of hard coding each step of the process; however, the

[33] Harari, 325.

resulting treatise of statistically generated rules for species counterpoint may be so immaculate that the endeavor is worth the rewards.

V. CONCLUSION

This thesis began as an exploration of specificity in the instructions given in Fux's treatise, *Gradus ad Parnassum.* While the experiment yielded interesting results regarding the necessity to prefer contrary motion above imperfect consonances, and the need for addenda on the treatise for concepts such as melodic climax and stepwise motion, the most interesting results began to suggest a connection between the strictness of these rules and the views of Heinrich Schenker on the fundamental structure of tonal music. By showing how precise adherence to sufficiently specific guidelines results in a single possibility for a contrapuntal melody, this experiment provides evidence to Schenker's view of tonal music as variations of a single underlying pattern. These findings are by no means conclusive, as the other four species have yet to be tested and the algorithms used here may be improved upon; however, the results given here are promising both for the existence of a fundamental structures and the use of computers as a means of discovering them. The methodology employed here can be adapted to many aspects of music theory—claims about harmonic construction and patterns, formal design, and even more modern techniques are susceptible to similar algorithmic tests. By utilizing computers in this way, music theory can be a much more rigorously scientific endeavor relying on data, evidence, and experimental design.

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