Learning Functions Through Vending Machines

Signature Honors Project

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by

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Abstract

Given the prevalence of function concepts in high school math classes, students need to develop a deep conceptual understanding of the topic. Technology tasks have become a frequent tool used in the high school classroom to teach functions. Technology is thought to enhance students' understanding of functions and foster a deeper understanding. The Vending Machine Task, through GeoGebra, provides students with various virtual vending machines to make selections and determine if each machine is a function or not. In this study, I have explored the effectiveness of technology centered tasks, such as the Vending Machine Task, and studied the strategies and academic language used while completing the task. My findings will show that the Vending Machine Task, and other technology tasks, expand students' thinking of functions and help build deeper conceptual understanding.

Introduction

Have you ever been to a vending machine, made your selection for the drink of your choice, and the machine dispensed two drinks? We always expect the machine to dispense exactly what we choose, so this outcome surprises us. However, what if the selection was meant to produce a different outcome than expected? As such, we can think of a vending machine as a metaphor to represent the mathematical concept of function, where your selection is the "input" and the product the machine dispenses is the "output". If the output isn't what we expect, is the machine still emulating a function? In this study, I will be researching how secondary mathematics students interact with and learn from virtual vending machines in a GeoGebra task. The Vending Machine Task challenges students to analyze the input and output of machines to determine whether it is a function or not.

As a preservice secondary math teacher, I take a notable interest in understanding how students work with and view functions. While spending time in local high school classrooms through observations and apprenticeships, I have seen how students develop misconceptions about functions and how that impacts other concepts within mathematics. Since I am approaching my internship and teaching career, knowing more information about functions in the secondary classroom will be invaluable. Similarly, functions make up a significant portion of the Common Core Mathematical Content Standards (National Governors Association, 2010). Throughout high school, students are expected to be able to interpret and build linear, quadratic, exponential, trigonometric, and rational functions. Given the prevalence of function concepts, it is crucial that students have a deep understanding of functions to be successful in advanced mathematics. Likewise, viewing how using technology in this task impacts students will help me learn best methods to use in my classroom. Through my research I hope to develop information that will allow teachers to better educate their students.

I believe the Vending Machine Task is appropriate to introduce students to functions. This applet, through GeoGebra, provides students with various vending machines for students to make selections and determine if the machine is a function or not. This task assists students with developing an understanding of relationships between quantities, comprehending input and output for a function, and creating a definition of function. All of these learning goals are prerequisites for the beginning function content standards for high school, such as determining rate of change and analyzing multiple representations. The Vending Machine Task also helps teachers implement mathematical teaching practices in their classrooms. Two teaching practices promoted by NCTM (2014) are *Implement Tasks that Promote Reasoning and Sense Making* and to *Use and Connect Mathematical Representations*. With this activity, students are required to make sense of and reason about the definition of function, as well as make connections from virtual vending machines to more traditional representations of functions. Using GeoGebra and other technologies allows students to have a more hands-on approach towards learning.

Accordingly, this research will explore the methods students use to complete the task. Seeing the methods that are used to analyze each machine will present myself and other educators with information about the development of functions. Also, I will analyze the common mathematical language used and promoted by this task. Understanding the language students use to process the Vending Machine Task will assist me with learning best practices to teach students function concepts. Therefore, my research questions are:

1. What are the most recurrent strategies or methods that high school students use to complete the Vending Machine Task?
2. What mathematical language do students use while completing the vending machine task?

Background Literature

When beginning to study the literature that exists around teaching functions in the secondary mathematics classroom, it is clear that this topic has been studied in a variety of ways over the last several decades. As we continue to progress through the many changes of the technology age, it is vitally important that students and teachers take advantage of the new mathematical opportunities. Each mathematical technology that is developed has the potential to shape mathematics instruction. Since I am studying how students interact with the Vending Machine Task and how it influences student thinking, I will be reviewing the literature around students' development and understanding of functions, the role of technology in assisting secondary students' understanding of functions, and the academic language that pertains to functions.

Secondary Students Development and Understanding of Functions

How Teaching and Tasks Impact Students Development and Understanding of Functions

The way that students begin to understand and develop the idea of functions begins in the classroom, specifically with the teacher. The way students are taught function concepts is the way they understand and interpret functions (Dubinsky & Wilson, 2013). When teaching functions, using multiple representations and demonstrating functions in varying ways benefits students. Initially, students are likely to have a limited view of representations of functions due to limited examples that are provided through instruction (Dubinsky & Wilson, 2013). Teachers are therefore tasked with expanding the way students think. It is known that when teachers spend time on teaching procedure and not the basic concept and definitions, secondary students are less likely to succeed in mathematics (Panaoura et al., 2016). Students best benefit when teachers lay down a strong foundation about functions, using multiple representations and various examples, and are able to build on function concepts easier in more advanced mathematics (Lovett et al., 2020).

The examples used during a function lesson are also equally important. According to Fernandez (2005), using real life examples allows students to think deeper about the topic and have a better understanding of the definition of a function and the concepts at hand. Examples provided in the classroom also influence concept images about a topic (Ayalon et al., 2017). For example, if teachers only show examples of functions represented on a graph, then students will not understand function through other representations and only have a surface level concept image. On the other hand, when students are exposed to numerous examples they will develop a deeper concept image. Students are more likely to learn more and interact with material better when they start with a familiar activity and build on to new concepts (Best & Bikner-Ahsbahs, 2017). Educators should therefore have flexibility in designing each task they are planning to assign to their students. Promoting tasks that challenge students' ways of thinking has also been shown as effective since students are pressed to study the topic in an alternate way (Carlson & Oehrtman, 2005). Ultimately, through tasks assigned by the instructor, students should be encouraged to work through functions in different representations. Tasks that promote deeper reasoning allows for the interchanging of ideas in the classroom and helps students have greater success with functions later on (Carlson & Oehrtman, 2005).

How Standards Influence Students' Development of Understanding Functions

Another factor that impacts the way students understand and develop functions is the standards. This is a factor that students and teachers have little control over, but must work to the best of their advantage. The way students view a function may rarely be similar to the mathematical definition, but that is likely due to the standards (Ayalon et al., 2017). When functions are first introduced in the standards, there is a limited exposure of the topic to students. For example, in the Math 1 standards, students are introduced to function notation and function composition algebraically, but do not analyze what this means graphically (National Governors Association, 2010). By learning this concept in isolation, students will struggle with making connections in the future. This causes students to see functions through a distorted outlook. Through the research conducted by Best and Bikner-Ahsbas (2017), this isolation of exposure regarding functions creates gaps when students learn advanced functions later in high school. Students understand material at the moment, but since they do not understand the concept of function it is more difficult to make connections throughout math courses. Over the years, curriculum has also been inconsistent with how and when functions are introduced. A common place of discontinuity among high school students appears to be their definition of function. Many students struggle with defining what a function is, which is a direct result of the isolation and inconsistency of functions as they appear in the mathematics curriculum (Leinhardt et al., 1990). While curriculum cannot be changed directly by educators, each classroom should strive to bridge the gap that exists amongst learning functions to create a deeper understanding in all students.

Students' Progression when Learning Functions

Levels of understanding functions vary among all students, but overall can be viewed as hierarchical (Ayalon et al., 2017). Students are thought to progress linearly though function concepts in secondary mathematics. It can be assumed that for students to link function concepts together and proceed with higher mathematics, they must have a deep conceptual understanding of initial concepts (Keller & Hirsch, 1998). Originally students should learn the basic definition and concepts of a function. Once this has been achieved, students are more capable of understanding properties of a function, such as one-to-one, onto, and inversion (Ayalon et al., 2017). While learning secondary mathematics, students should ultimately be able to compare and make judgements of functions in order to promote a deeper understanding of higher mathematics (Carlson & Oehrtman, 2005). The examples and experience that students are exposed to do impact their learning of functions.

The Role of Technology in Developing Students' Understanding of Functions

Technology and Students' Understanding of Function

While teachers do have the task of demonstrating functions through a variety of representations, technology helps make the task less burdensome. Using technology to show functions in a variety of ways yields deeper and more flexible understanding in students (Keller & Hirsch, 1998). When students have the opportunity to explore and visualize functions, they are likely to be more successful in applying that knowledge to other questions. Breidenbach et al. (1992) observed the impact that computers have had on students' learning. Students showed strong improvement when using a computer and having visual manipulations as opposed to just using paper and pencil tasks. Graphing calculators have also been shown as a useful tool in learning functions. When students were given calculators to complete a function assignment, they tended not to rely on a preferred representation, but had a deeper understanding of each representation that was given on the task (Keller & Hirsch, 1998). As technology continues to develop, it is important for teachers to find ways to strategically incorporate technology because research supports that such use enhances students' mathematical thinking and understanding (e.g., Burrill et al., 2002; Zbiek et al., 2007; Heid & Blume, 2008; Hollebrands & Dove, 2011).

The Vending Machine Task and Students' Understanding of Functions

The Vending Machine Task has the potential to help students develop their understanding of functions because it helps students move away from focusing on procedural methods, such as the vertical line test, to being able to complete more complex problems (Lovett et al., 2020). Lovett et al. (2020) utilized this task in a middle school classroom, shortly after the class had been introduced to functions. Students were grouped in pairs and were given an opportunity to work through each machine multiple times, then each group constructed a definition and tested their conjecture against the vending machines. At the start of the task, students were focusing on the "correct" output from the vending machines, but later began to focus on the consistency of what the machines produced (Lovett et al., 2020). These findings show how middle school students were impacted by this task and the technology they used, but the impacts this task would have on secondary students remain unknown. Given that I am a preservice secondary mathematics teacher, I am interested to see how high school students interact with and benefit from a Vending Machine Task designed for the high school level.

Academic Language and Defining a Function

Mathematical language can be difficult for some students to understand or make sense of. Some students do not even want to provide a definition of a function because they are afraid it will not be formal enough, or would use the wrong vocabulary language (Panaoura et al., 2016). It should be the goal of each teacher to have every student comfortable enough with the mathematical language to be successful with any function task. In research conducted by Fernandez (2005), it was shown that after students were able to identify the domain and range of a relation, they were typically able to identify if the relation is a function. This research makes clear the importance of helping students become comfortable with the academic language so they can be more successful in mathematics.

Oftentimes, students struggle with writing definitions of functions. When students are asked to create these definitions, they often do not contain function theory and vocabulary in a way that a formal definition would. Instead, these generated definitions focus more on the exposure the students have experienced with functions (Panaoura et al., 2016). The vertical line test also seems to be a stumbling block for most students. The vertical line test states that if one draws a vertical line on a graph and the line passes through the given image more than once it is not a function. Most students view this idea as a definition, but cannot explain the underlying concepts (Dubinsky & Wilson, 2013). Teaching the vertical line test can be a benefit to students, but also can be a hindrance. Once students learn the test, they often tend to disregard other aspects of functions (Fernandez, 2005). This research displays the importance of teachers using academic language in the classroom and allowing students to develop definitions regarding functions.

The Vending Machine Task as a support for Academic Language and Defining Function

In the Vending Machine Task that was completed by middle school students, as described above, each pair created their own definition of a function. Before students began working independently in their groups, the class as a whole came to an agreement about what the input and output were for the first machine (Lovett et al., 2020). By having the class come to an agreement about the machines they were working with, each student began the task with an understanding of how the machines worked and what role each piece played throughout the process. Next, students began working in their groups to conjecture their idea of a definition of a function. Students focused on terms such as rules, proportional, and randomness (Lovett et al., 2020). These are the same terms that seemed to be recurrent throughout the task. Having this language at hand benefited students with working through the machines and understanding the task better.

Ultimately, previous research has shown the importance of choosing tasks that promote thinking and understanding of functions for students. Teachers play a critical role in how students view and talk about functions. Given that functions are a large concept for students to learn, studying how students interact with particular tasks would be beneficial for the education community. The Vending Machine task is an unconventional way to think about functions. This medium moves students from thinking about the procedural steps they have learned previously and helps with developing a rich conceptual understanding of functions.

Methods

When beginning to conduct research using the Vending Machine Task, it was critical to consider what students will best benefit the education and research community. Given that research has already been studied using this task among middle school students, it would likely have been unconstructive to repeat this with that group of students. Ultimately, it was decided that secondary, or high school students, would be the participants in this study. The participants in this study are high school students that have agreed to complete this task and have their responses used. The videos are a part of a larger project Preparing to Teach Math with Technology-Examining Student Practice (PTMT-ESP) funded by the National Science Foundation (DUE 1820967). For the purposes of this study, five videos were selected to be evaluated regarding students' strategies for working through the vending machine task and the academic language used throughout the activity.

The Vending Machine Task

Since videos of student interactions with the Vending Machine Task and associated student work were the primary focus, this research will be qualitative in nature. In the education field, it is common for researchers to conduct qualitative research due to extensive use of student artifacts and teaching analysis (Bogden & Biklen, 2007). Data was collected in two southeastern states by two high school math teachers during a whole class lesson featuring the Vending Machine Applet. The applet was built in GeoGebra (GGB) and consisted of 4 pages of vending machines for students to compare if they are function or non-function. On the first three pages there were two machines on each page and students had to determine which machine was a function and which one was not a function (see Figure 1). The last page contained 6 vending machines for students to analyze individually to determine if they are functions or not (see Figure 2).

Figure 1

Vending Machine Page 1

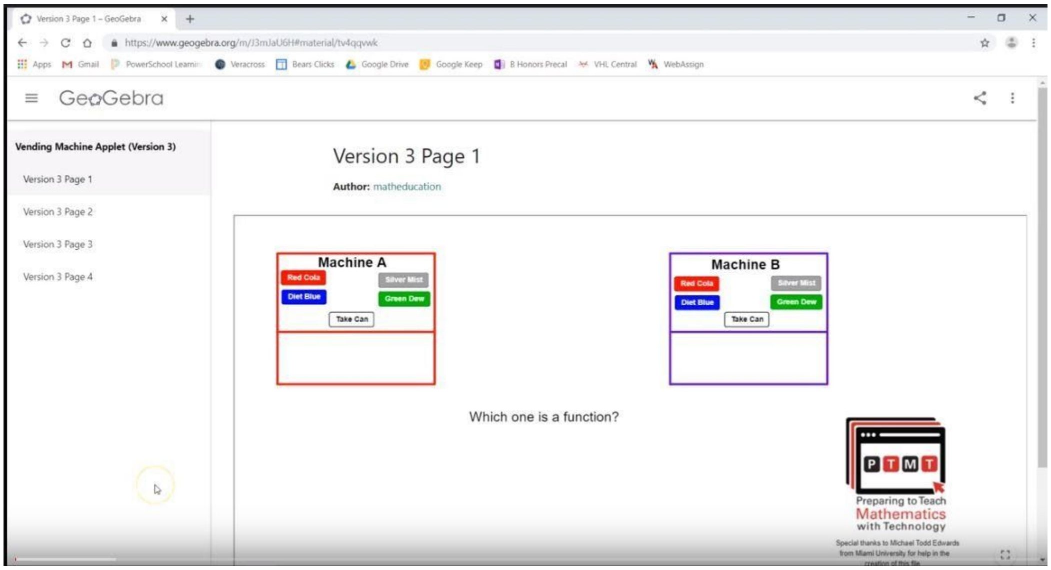
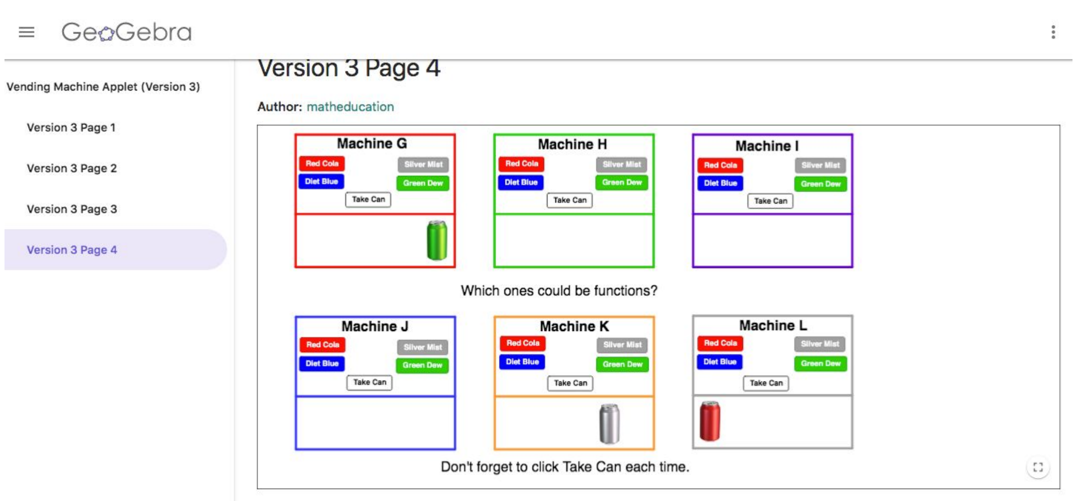


Figure 2

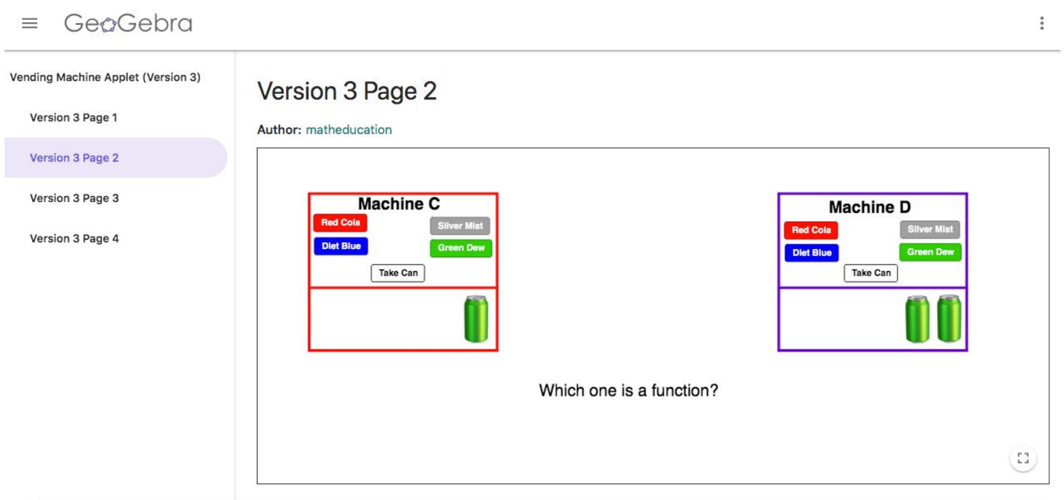
Vending Machine Last Page



Each vending machine has four colors: red, blue, silver, and green. Each selection produces a "can" out of the machine (see Figure 3). Once a "can" is dispensed, students must then click "Take Can" before making the next selection. Groups were reminded by their teacher at the start of the task to use the "Take Can" button, yet some groups still did not follow that

Figure 3

Vending Machine Page 2



instruction. In some machines, the color that is selected will give the corresponding can, but in others it will give a different colored can. As students continue to work through the task, they may notice that some selections produce two cans (i.e., Machine K), no can at all (i.e., Machine J), or a random colored can each time (i.e., Machine G). Throughout the progression of the task, the machine outputs vary to lead students to think about what it means to be a function and expand their thinking. At first students will be exposed to outputs (same color, single can) that correspond with the input (color of can selected), but eventually the outputs will all vary (e.g., single can, different color than selection). Students will also begin with one can as the output and then see machines that dispense two cans at once. When students make their selections there are different strategies that can be used to test out the machines to determine if they are functions or not. For example, students could vary the way they select cans, take the can each time, and vary how they compare machines to one another. There is also different language that can be used to work through the machines themselves. Students could use words such as input and output, x- and y- values, or compare machines to other representations.

Coding of the Vending Machine Task

Before analyzing videos of students engaging with the task, there were some predetermined, or a priori codes that were constructed (Miles and Huberman, 1994). The researcher first worked through the task to determine different methods that could be used to solve the task. Once reviewing the task and other research that has been conducted about the Vending Machine Task from Lovett, et al. (2020), the researcher finalized the a priori codes to be used for students' strategies (Table 1) and academic language (Table 2) .

For strategies, the a priori codes were:

Table 1

A Priori Codes for Research Question 1

Code Letter	Code	Example
O	Students only click each color one time	A student clicks red, blue, silver, and green once in any order.
M	Students click each color for the machine once in order and repeat multiple times	A student clicks red, blue, silver, and green in order and cycles through multiple times.
A	Students click each color multiple times, before moving to the next color	Red is clicked three times, then blue three times, silver three times, and green three times.

For student language, the a priori codes were:

Table 2

A Priori Codes for Research Question 2

Code Letter	Code	Example

C	Students identify that the output from the color selection changes, or varies each time.	Students click red multiple times and say that the output is different; i.e. it produced red the first time, but green the second and third time.
S	Students notice that the selection produces the same, or consistent, can each time.	When silver is clicked, the student says that silver always gives an output of green
O	Students state that the selection only produces one can, or one value, when selected.	Students say that every time the color red is selected it only gives an output of one can.
M	Students identify that the selection produces multiple cans, or multiple values, when selected	Every time the color silver is clicked, students state that both the color red and silver are produced.

After determining the a priori codes, the researcher and faculty mentor proceeded with analyzing the first video using these codes independently. Throughout the analysis, the list of codes expanded to include emergent codes (Stemler, 2000) that were found by the researcher and faculty mentor as follows:

For strategies, the emergent codes were:

Table 3

Emergent Codes for Research Question 1

Code Letter	Code	Example
T	Students clicked take can after each selection	Each time a can is selected, the student clicks take can.
D	Students did not click take can after each selection	A student selected red and blue one after the other without taking the can.
C	Students only clicked one color multiple times, other three colors only once	The red can is clicked multiple times in a row, but the other colors are only clicked once.

W	Make function/non-function decision without testing other machines.	Students work through Machine A and decide it is a function without testing Machine B.
N	Make function/non-function decision by comparing or contrasting with other machines	Students work through Machine H, but compare it to Machine C to decide if it is a function.

For student language, the emerging codes were:

Table 4

Emergent Codes for Research Question 2

Code Letter	Code	Example
I	Students discuss the input, or x-values, when referring to the selection.	The selection clicked is referred to as the input or x-value when discussing the machines.
Y	Students mention the output, or y-values, when discussing the product.	Students call the can that was produced the output or y-value when talking about what the machine gave.
R	Instances when students relate what they have seen from the machine to the graph of a function or non-function or a function itself.	When students notice that each selection produces the corresponding color, they compare the machine to a line.
E	Instances when students relate what they have seen from the machine to the equation of a function or non-function or a function itself.	When students notice that each selection produces the correct color, they verbalize that the equation of the machine is x = y.
T	Instances when students relate what they have seen from the machine to the table of a function or non-function or a function itself.	Students assign each color a number and create a table using ordered pairs to analyze the machine.

While coding each of the videos, multiple codes could be assigned to one pair of students. For example, if a student only clicked each color one time (Code O), but also clicked the take can button after each time (Code T). The pair of students would be assigned both codes. If two codes happened at the same time, they were recorded with a comma. So if the students clicked each color once and clicked take can each time, it would be recorded as O,T. Likewise if students clicked each color only once (Code O) but then went back to the machine and then selected each color multiple times in a sequence (Code M), it would be recorded separately since it happened in a sequence.

After both the research and faculty mentor had completed analysis of the first video, they compared their coding and reviewed any discrepancies and emergent codes to revise the overall coding definitions (see appendix A) to use with the remaining videos. This resulted in eight codes being used for strategies and nine codes being used for language. Each video was coded independently for each research question. The researchers coded for strategies first, then coded for language separately afterwards. Next, the second video was coded independently by the researcher and the faculty mentor. After the coding was complete, no additional codes were noted and a 85% inter-rater reliability was achieved. The final three videos were then analyzed by the researcher. Codes were completed based on student actions and language spoken with their peers. For example, when considering student strategies, if the researcher observed a student not clicking the take can button after each time the emergent code D was assigned to the group. If the students later began to click take can after each selection on the same machine that would also be assigned a code of T. For student language, if a student was explaining their selection by describing the graph of a similar function, the researcher assigned the group a code of R. The codes were then compiled to be analyzed.

Results

The purpose of this study is to understand how students view and understand the concept of functions. This topic was understood by answering the following research questions: "What are the most recurrent strategies that high school students use to complete the Vending Machine Task?" and "What mathematical language do students use while completing the Vending Machine Task?". After viewing videos and analyzing student work, the researcher compiled the results for each research question. The results for research question one are summarized in Table 5, and the results for academic language are summarized in Table 6. For a complete list of codes see Appendix A.

Student Strategies (RQ1)

In the table below, codes are shown for student strategies. If students used two codes at the same time, such as the way the cans were selected and then taken from the machine, they are represented with a comma. If the codes happened in a sequence they are listed as one entry after the other.

Research Question 1:

Table 5

Research Question 1 Group Codes

Machine	Group 1	Group 2	Group 3	Group 4	Group 5
A & B	O,T	O	O	O,T	O,T
	M,T	D	T	C	C
	M,D	N	M,D	N	N
	N		N		

C & D	M T W	D,M W A,D O,T	O D M,D N	O,T A,T N	M T A,D N
E & F	M T N	D A C D,T M,N	M,T N M,D	M,T N	A D N
G	O T W	T M,A W	M T W	O T W	A D M,W
H	M T W	O A C N,T	M T W	O T W	A D M,W
I	M T W	M T W	A T W	O T W	A D W
J	M T W	T M W	M T N	M,T W	A D M,T W
K	O,T C,T W	T M W	M N,T A,T	M,T N	A D W
L	O T M W	T M W	M T W	M,T N	A,D M,T W

When first beginning to interact with the Vending Machine Task, it was evident that nearly all students had never seen functions through this representation before. For the first couple of machines, A/B and C/D, all groups began by only working through the can selections a single time (Code C). Through trial and error or by clicking a color twice accidentally, three out of the five groups quickly realized that they must select each color multiple times. Once each group came to that understanding, patterns began to emerge for how to select the colors. Four groups (groups 1-4) consistently selected the colors one after the other in order and repeated the sequence over again (Code M), while one group (group 5) clicked one color multiple times in a row before moving to the next selection (Code A).

The issue of taking the can after each selection was also prevalent towards the beginning of each groups' interaction. After the first few machines, at least four of the five groups noticed that if they did not click the take can button the cans would pile up and they would not be able to accurately see the outputs. Groups 1-4 began to take the can after every selection (Code T) beginning around Machine E/F, however, Group 5 did not click the take can button throughout the interaction (Code D). Group 2, after noting that the take can button needed to be selected each time, chose to revisit the first couple of machines to verify their answers.

Throughout all tabs on the task, while groups were deciding if each machine was a function or not, the researcher focused on how the students compared the machines to one another to arrive at their decisions. Towards the start of each groups' interaction, two to three groups repeatedly did not compare the machines to one another and solely reached a conclusion by what they were seeing from the input and output (Code W). The remaining groups consistently compared machines to one another for their similarities, differences, or to justify their answers with their peers (Code N), as shown from Group 1 when interacting with machines A and B:

Student 1: "I don't think B can be a function since two different x-values give the same y-value. See, when I click green it gives me silver and when I click silver I also get silver."

Student 2: Yeah, that makes sense because on A every x-value has different y-values so it has to be a function. Since B had the same y-values then it is not a function."

Once groups reached the final page, which contained Machines G-L, four out of five groups consistently did not compare the machines with one another before making their decision, as shown from Group 5 when interacting with machine K:

Student 1: "When I click green I get both green and red cans."

Student 2: "Well since it is consistent each time and it produces the same thing, even though it is wrong, it would still be a function. Right?"

Student 1: "I think so, it always would give the same result so it has to be a function."

Academic Language (RQ2)

In the table below, codes are shown for student language. If students used language for two codes at the same time, such as using input and output together, they are represented with a comma. If the codes happened in a sequence they are listed as one entry after the other.

Research Question 2:

Table 6

Research Question 2 Group Codes

Machine	Group 1	Group 2	Group 3	Group 4	Group 5
A & B	I,Y C,S R	C S	S C	C,Y S	C S
C & D	O	S,C	S	I,Y	M

	M I,Y R	O,M Y,I R,E	C	M S	C S
E & F	O M O,Y	S M R I,Y E	C,M S O	C M Y	M C S
G	R C	R,C I,Y	C	I Y	C S
H	R I,Y E	S R	S	Y	S
I	M O	M,O R,E Y,I		M Y	M S
J	R	R O	I,Y C	M I,Y	S Y E
K	M Y	M	S,O M	C Y	M C,S
L	R E	R E I,Y	S O	I,Y T	S

While working through the task, each machine invoked students to use language with their partners to denote the consistency, or lack thereof, for each machine. Students quickly realized, all groups on Machine A/B, that some selections would consistently produce the same results (Code C) while others would produce something different each time (Code S). This observation was one that students would typically express first when clicking through the color selections. Starting on Machine C/D, all groups also began to express the quantity of cans that each selection produced. Some selections would produce two cans (Code M), consistent or inconsistent in color, while others would just produce one (Code O). Students then would note the quantity and consistency of color outputs when determining if a machine was a function or not.

Since the Vending Machine Applet was new for students, the researcher focused on their language in regard to connecting the representation to things they were familiar with. Groups began to make the connections by using words such as input, or x-value (Code I) and output, or y-value (Code Y). Group 1 immediately began this connection by describing the color selection as the input and the can that was produced as the output on Machine A/B. Two other groups did the same for Machine C/D, but by the end of the interaction, all groups were using the input/output language in different forms.

Another strategy that students used to understand the machines was to compare the machines to graphs (Code R), equations (Code E), or tables (Code T). These representations are common for students to use when learning about functions, so some students chose to transform the machines to representations that were more familiar. Groups 1 and 2 were the more frequent groups to make these comparisons and would typically do so when trying to understand what the outputs meant or explaining their thinking to their peers. These groups often made multiple comparisons for machines on each page. Graphs were more common since groups were able to visualize what the graph would look like and think about the vertical line test and other techniques they had learned about knowing if a graph is a function or not. Group 4, however, compared the vending machine to a table to understand how the function changed for each selection.

Discussion

Through analyzing the results, it was clear that the Vending Machine Task was a new tool for students. Because of this, there was often a learning curve for groups to understand and properly use the representation. For example, all of the groups were unsure of how to test the machines and Group 5 did not use the "Take Can" button adequately. While this learning curve did make using the task more challenging for students to interact with, it also provided a clear layout of how students think about and understand functions. This task required students to demonstrate their deeper understandings of function concepts and removed any procedural knowledge that centered around familiar representations. The strategies and mathematical language that students used demonstrate how this task can be beneficial when teaching functions.

The varying strategies that students used to test each machine impact their understanding of functions. At the beginning of the task, all groups began by only choosing each selection once. After realizing the need to check each selection multiple times, Groups 1-4 selected the colors in order multiple times while Group 5 selected each color multiple times in a row. Research has shown that students typically rely on the vertical line test to determine if a relation is a function or not (Dubinsky & Wilson, 2013). The method of selecting colors one after the other in a sequence moves students beyond thinking about function graphically. Students are then moved to notice patterns about input and output and not think about each input individually. This supported students' conceptual understanding without using algebraic representations (Lovett, et al., 2020). Group 5, which selected each color multiple times in a row, did continue to focus on each input in isolation, but did expand beyond their knowledge of graphical representations. Either method shows that this task promotes deeper thinking among students and moves students away from their procedural techniques that they have previously learned.

Students' use of the "Take Can" button was also critical in their understanding of what it means for each machine to be a function or not. Some groups progressively realized the importance of taking the can each time, while Group 5 consistently did not take the can during the task. Group 2 realized the importance about halfway through the task and then chose to revisit the machines they had already determined. This revealed that if the can was not taken each time the machines would not demonstrate a clear picture of what each input produced. This concept allowed students to understand how each input can have the same outputs or that different inputs can produce the same output. These ideas would not likely be evident to students when working with representations they are familiar with, so this task brings students to this realization. By moving students to different representations, they are more likely to develop a deeper understanding and make connections to the more familiar representations (Lovett, et al., 2020).

Research has shown that students often have incorrect or limited definitions of function or function concepts, likely due to their limited exposure to functions (Panaoura et al., 2016). This task supports the research that students struggle with constructing meaningful function definitions, but it also assists them with making connections to build definitions. For example, students were using language about input and output, which they were likely familiar with and had a limited understanding of. However, students were also using language about consistency, multiple outputs being produced, and repetition. The language this task developed in students will likely allow them to form deeper definitions about function concepts. Students were also able connect their previous thinking about functions to their new discoveries. For example, Group 1 consistently compared machines to graphs, tables, and equations that they were previously familiar with. These connections allow students to develop a conceptual understanding of functions and make connections from their procedural knowledge (Dubinsky & Wilson, 2013).

While this task alone would not be sufficient to teach students functions, this task would likely be beneficial to build students' early concepts and ideas about functions on the secondary level. This task provides students and teachers with opportunities to build procedural fluency from conceptual understanding instead of focusing on procedures (NCTM, 2014). Students and teachers would benefit from using this task in the secondary classroom. Students need a strong foundation of function concepts to be successful in their higher level mathematics courses and develop strong mathematical reasoning skills. Students move from thinking about functions procedurally to more abstractly by considering a new representation. This task helps students make connections between previously learned function ideas to new concepts to build a deeper understanding and build stronger mathematical language skills to discuss and understand functions. This task has also influenced the way I think about teaching function in the high school classroom. I have seen first hand that once students determine the procedural method for finding a solution they are less likely to develop conceptual understanding. This task moves students to understand the concept first and then build the procedures. Overall, the Vending Machine Task would be an effective tool to use in the secondary math classroom.

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Appendix A

Strategy Codes

A priori codes/pre-set (codes identified to use before analysis):

- **O :** Students only click each color one time
- **M :** Students click each color for the machine once in order and repeat multiple times
- **A:** Students click each color multiple times, before moving to the next color

Emergent codes/come up as analyzing video/artifacts:

- **T :** Students clicked take can after each selection
- **D :** Students did not click take can after each selection
- **C :** Students only clicked one color multiple times, other three colors only once
- **W:** Make function/non-function decision without testing other machines
- **N:** Make function/non-function decision by compare/contrast with other machines

Language Codes

A priori codes/pre-set (codes identified to use before analysis):

- **C** : Students identify that the output from the color selection changes, or varies, each time. Each y-value, or output is different
- **S** : Students notice that the selection produces the same, or a consistent, can each time. Each y-value, or output is the same
- **O** : Students state that the selection only produces one can, or one value, when selected
- **M** : Students identify that the selection produces multiple cans, or multiple values, when selected

Emergent codes/come up as analyzing video/artifacts:

- **I** : Students discuss the input, or x-values, when referring to their selection
- **Y** : Students mention the output, or y-values, when discussing the product
- **R** : Instances when students relate what they have seen from the machine to the graph of a function or non-function or a function itself
- **E:** Instances when students relate what they have seen from the machine to the equation of a function or non-function or a function itself
- **T**: Instances when students relate what they have seen from the machine to the table of a function or non-function or a function itself